Design of crystal oscillators

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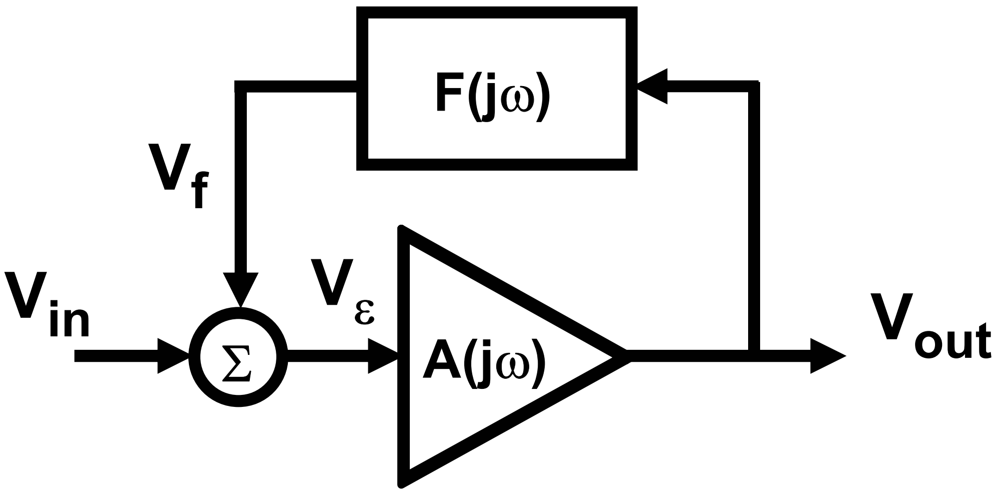
The Barkhausen criterion



$$V_{out} = A(j\omega)\, V_\varepsilon$$

$$V_f = F(j\omega)\, V_{out} = F(j\omega)\, A(j\omega)\, V_\varepsilon$$

$$\frac{V_f}{V_\varepsilon} = A(j\omega)\, F(j\omega)$$

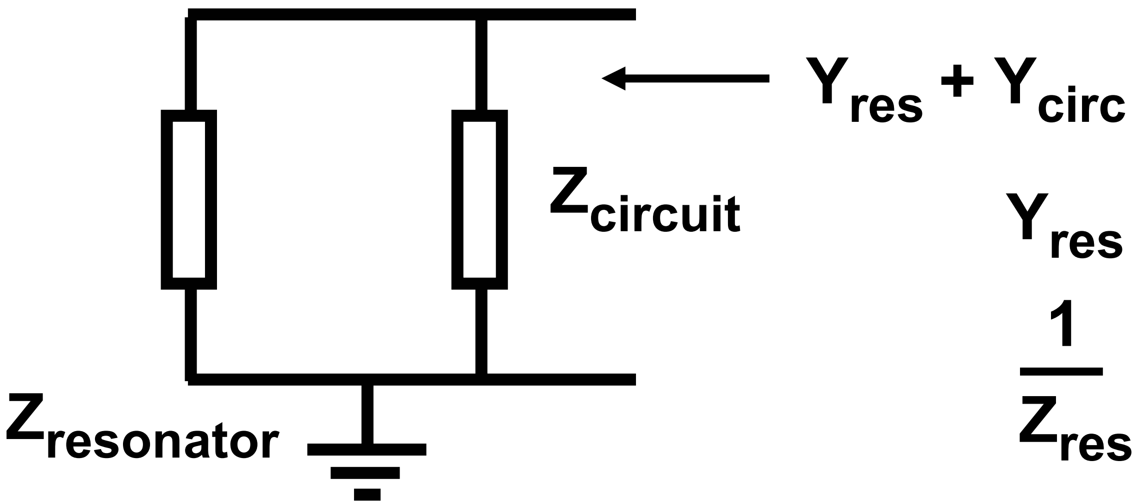
Oscillation if $V_{in} = 0$ or if $\left|\frac{V_f}{V_\varepsilon}\right| = |A(j\omega)|\ |F(j\omega)| \geq 1.0$

Positive FB !

$$\left\{\frac{V_f}{V_\varepsilon}\right\} = \Phi_A + \Phi_F = 0^\circ$$

Ref. Barkhausen, Hirzel, Leipzig, 1935

Split analysis



$$Y_{res} + Y_{circ} = 0$$

$$\frac{1}{Z_{res}} + \frac{1}{Z_{cir}} = 0$$

$$\frac{Z_{circ} + Z_{res}}{Z_{res}\, Z_{circ}} = 0$$

Oscillation if $\mathrm{Re}\,(Z_{circ}+Z_{res}) = 0$ **sets the minimum gain !**

$\mathrm{Im}\,(Z_{circ}+Z_{res}) = 0$ **sets the frequency !**

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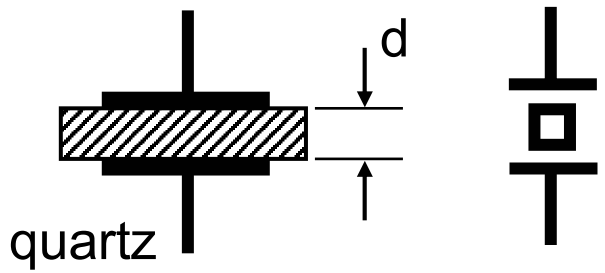
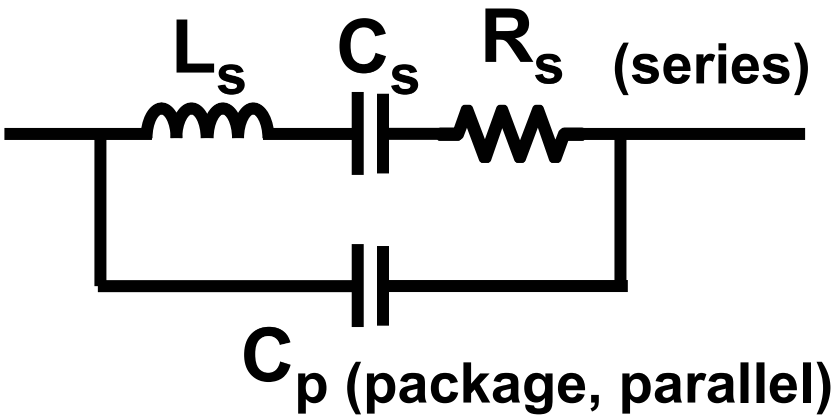
Crystal as resonator



$$f_s = \frac{1.66}{d}$$ f_s in MHz if d in mm

$$C_p = A\,\frac{\varepsilon_0\,\varepsilon_r}{d}$$ $\varepsilon_r \approx 4.5$

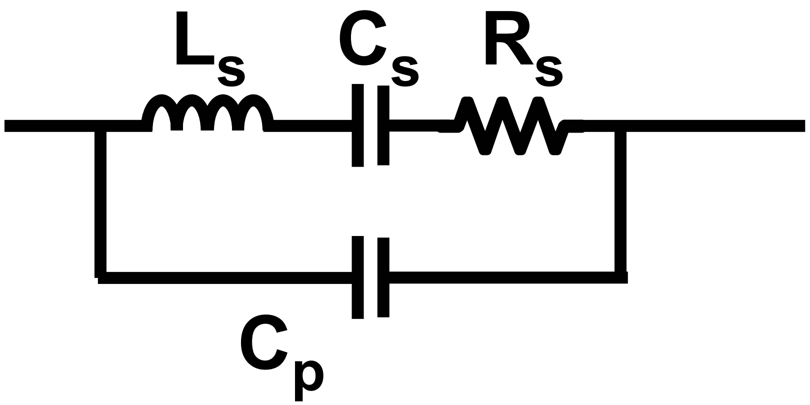


$$\omega_s^2 = \frac{1}{L_s\,C_s} \qquad f_s = \frac{1}{2\pi\sqrt{L_s\,C_s}}$$

$$L_s\,\omega_s = \frac{1}{C_s\omega_s} \qquad Q\,\omega_s = \frac{1}{R_s C_s}$$

$$Q = \frac{1}{R_s}\sqrt{\frac{L_s}{C_s}} \qquad R_s = \frac{1}{Q\,C_s\omega_s}$$

Crystal parameters



Xtal : f_s = 10.000 MHz

$Q = 10^5$

C_s = 0.03 pF

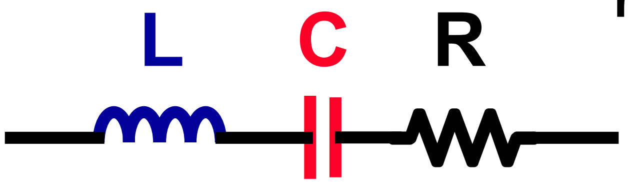
$C_p \approx$ 6 pF ($\approx$ 200 C_s)

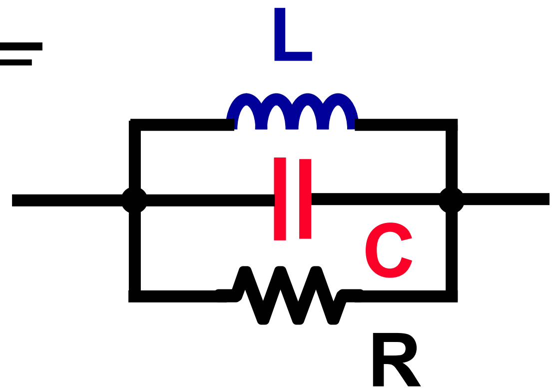
$$L_s\,\omega_s = \frac{1}{C_s \omega_s} \qquad L_s \approx 8.4 \text{ mH} \qquad R_s = \frac{1}{Q\, C_s \omega_s} = 5.3\ \Omega$$

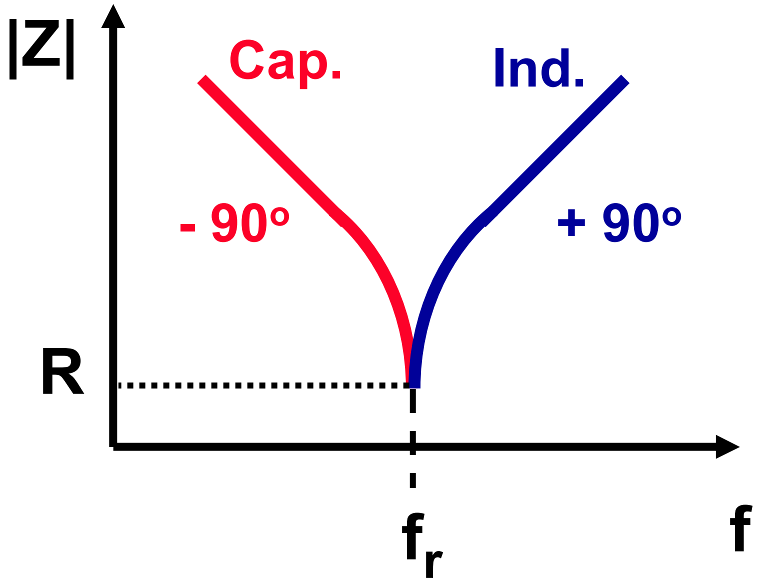
f_s	L_s	C_s	R_s	C_p	Q
100.0 kHz	52 H	49 fF	400 Ω	8 pF	$0.8\ 10^5$
1.000 MHz	2 H	6 fF	24 Ω	3.4 pF	$5.3\ 10^5$
10.00 MHz	10 mH	26 fF	5 Ω	8.5 pF	$1.2\ 10^5$

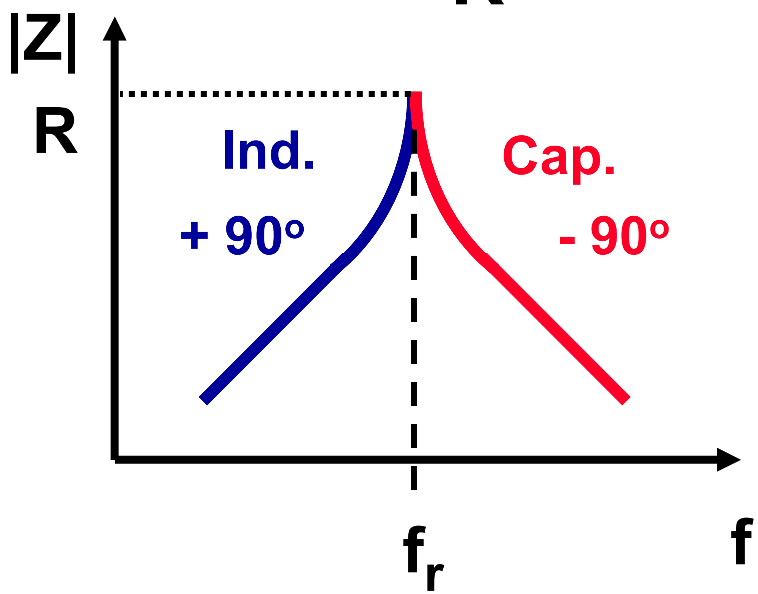
Series and parallel resonance

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$



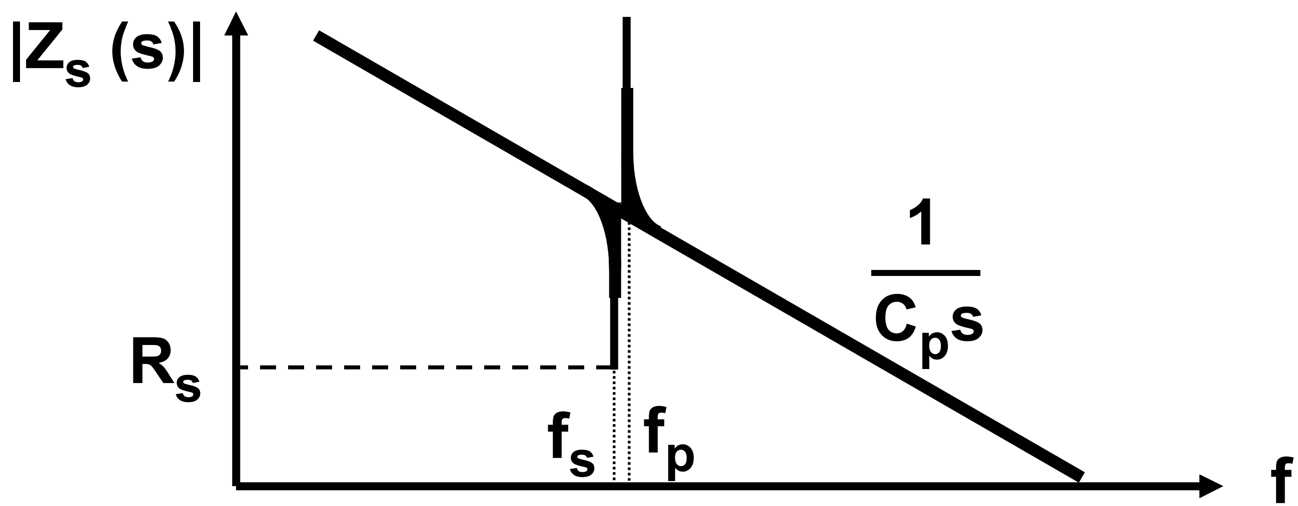




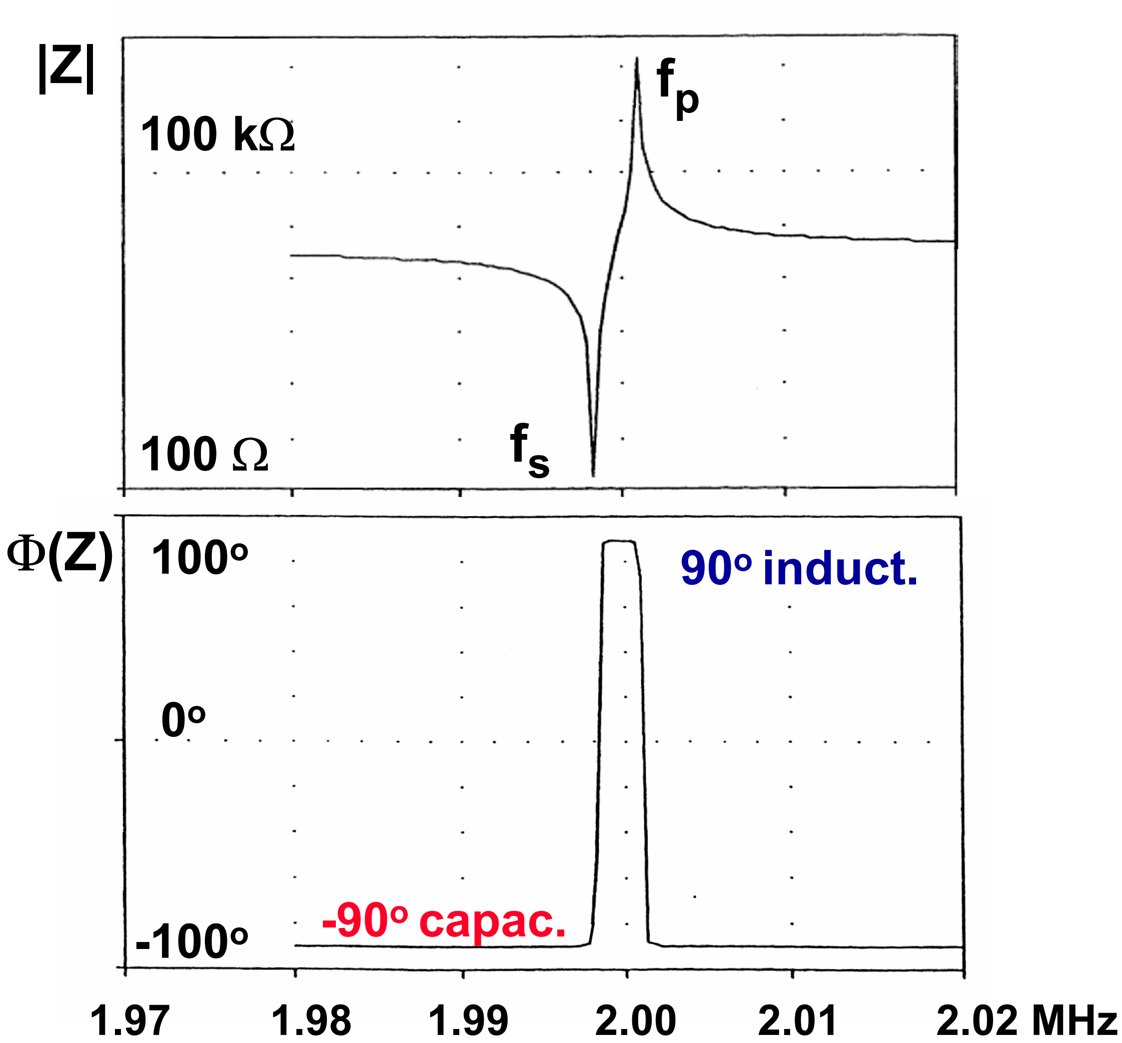


Crystal impedance

$$Z_s(s) = \frac{s^2 L_s C_s + s R_s C_s + 1}{s\,(C_s + C_p)\left(s^2 \dfrac{L_s C_s C_p}{C_s + C_p} + s\,\dfrac{R_s C_s C_p}{C_s + C_p} + 1\right)}$$



Crystal impedance at resonance



f_s = 1.998 MHz

C_s = 12.2 fF

L_s ≈ 0.52 H

C_p = 4.27 pF

R_s = 82 Ω

Crystal operates in
inductive region
if circuit is capacitive !

Series and parallel resonance

$$Z_s(\omega) = \frac{-j}{\omega C_p} \frac{\omega^2 - \omega_s^2}{\omega^2 - \omega_p^2}$$

$$\omega_s^2 = \frac{1}{L_s C_s}$$

series

$$\omega_p^2 = \frac{1}{L_s}\left(\frac{1}{C_p} + \frac{1}{C_s}\right)$$

parallel

$$Z_s(\omega) = R_s + j\omega L_s + \frac{1}{j\omega C_s}$$

$$Z_s(\omega) = R_s + \frac{j}{\omega_s C_s}\left(\frac{\omega}{\omega_s} - \frac{\omega_s}{\omega}\right)$$

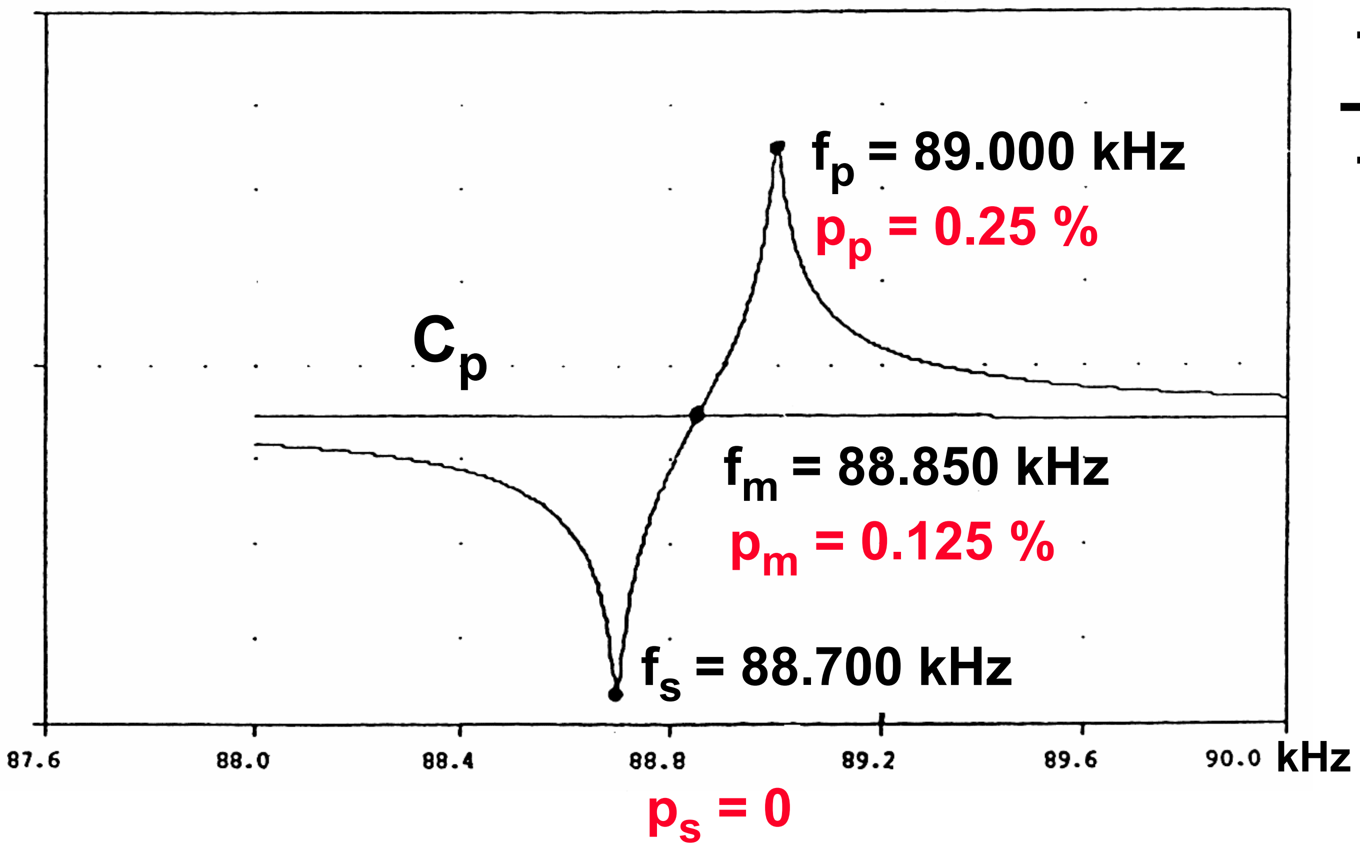
$$Z_s(\omega) \approx R_s + j\frac{2p}{\omega C_s}$$

Frequency pulling factor

$$p = \frac{\omega - \omega_s}{\omega_s}$$

Ref. Vittoz, JSSC June 88, 774-783

Series or parallel resonance ?



$$\frac{f_p}{f_s} = \sqrt{1 + \frac{C_s}{C_p}}$$

$$\approx 1 + \frac{C_s}{2C_p}$$

$$p_p = \frac{f_p - f_s}{f_s}$$

$$\frac{C_s}{2C_p} = 0.25\ \%$$

$$p_m = \frac{f_m - f_s}{f_s} = \frac{C_s}{4C_p} = 0.125\ \%$$

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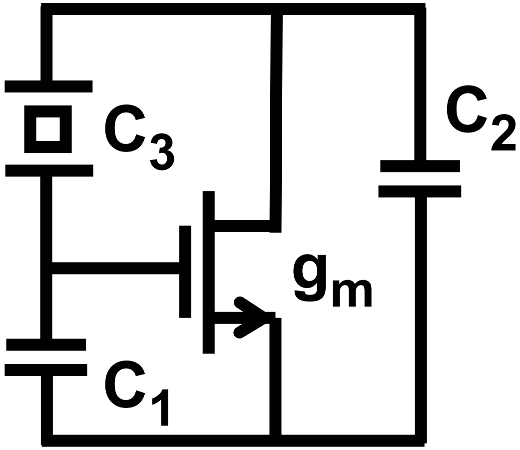
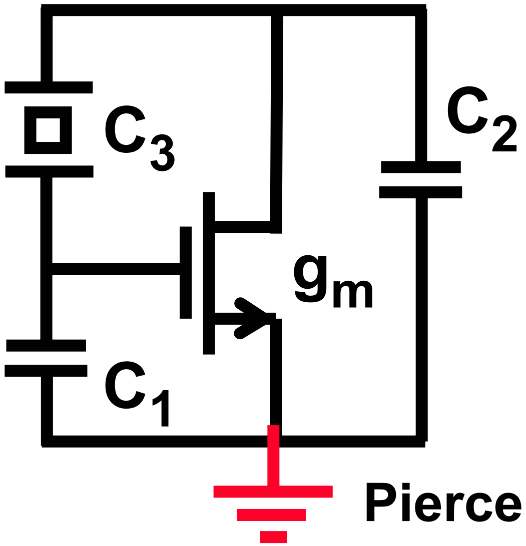
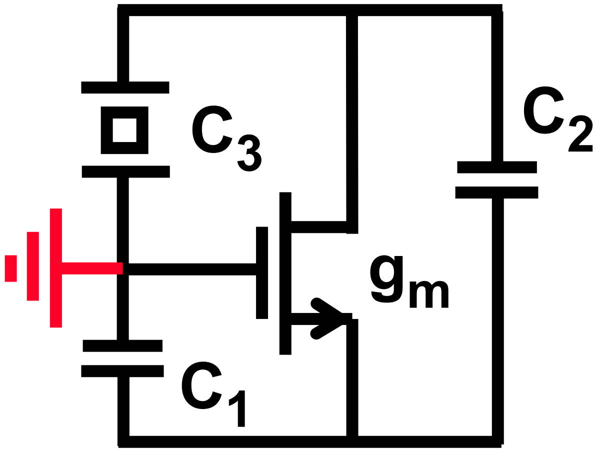
Single-transistor X-tal oscillator



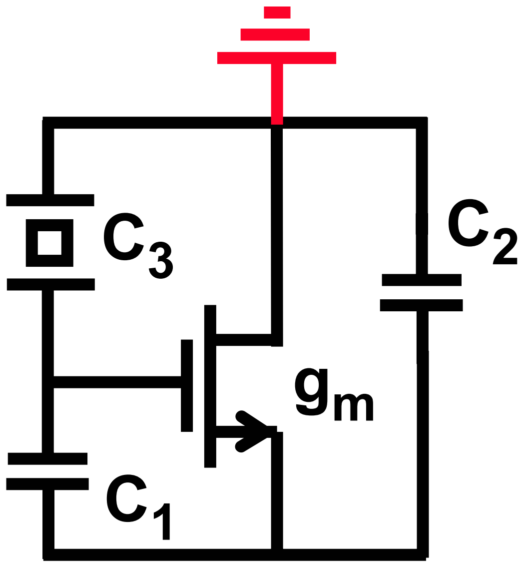
Basic three-point oscillator



Pierce

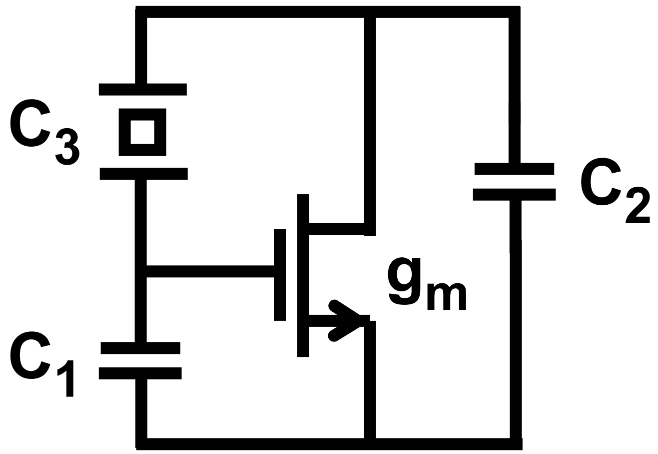


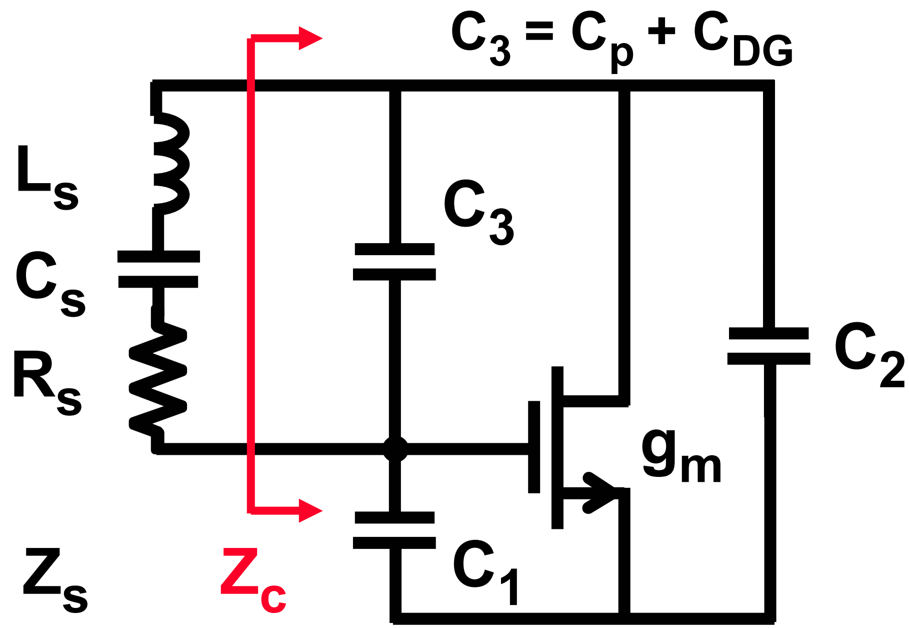
Colpitts : 1-pin X=D



Santos : 1-pin X=G

Single-transistor X-tal oscillator analysis





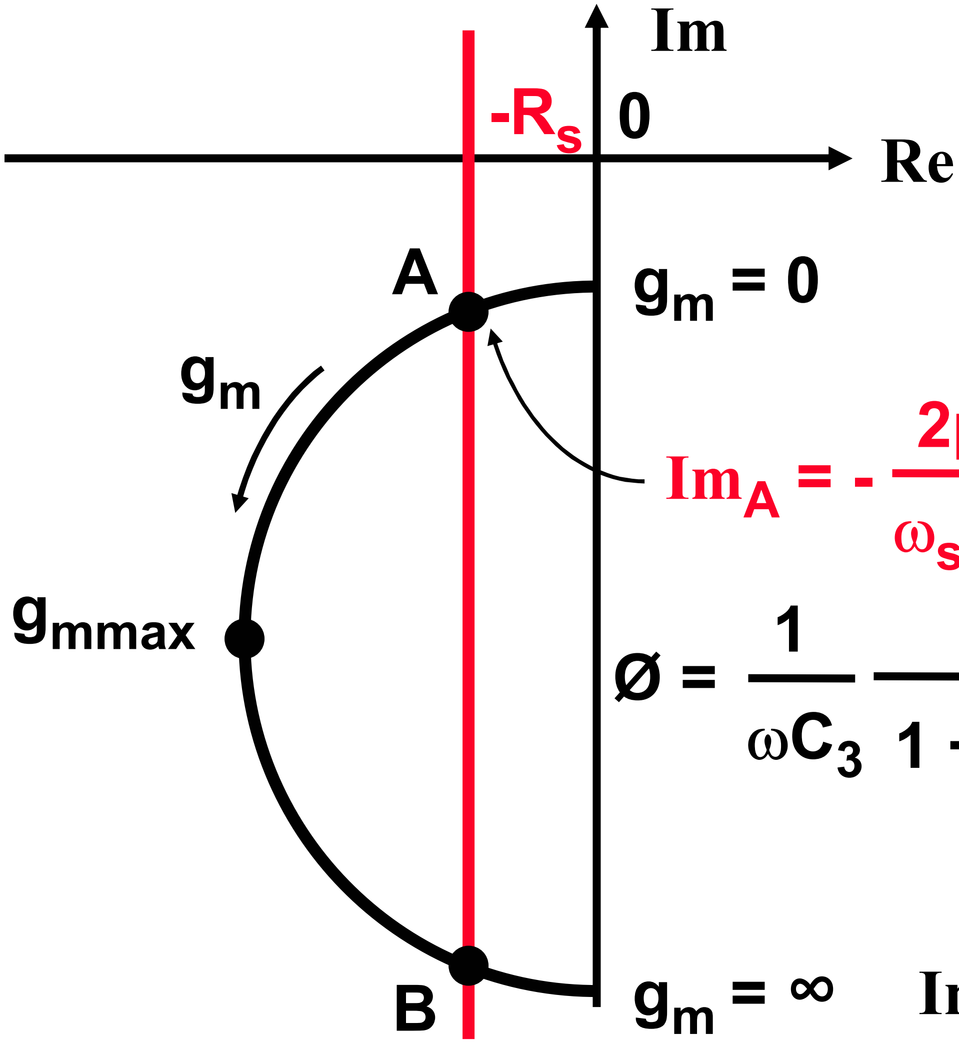
Barkhausen : $Z_s + Z_c = 0$ $\quad$ $Z_s = R_s + j \dfrac{2p}{\omega C_s}$

$Re(Z_c) = -R_s$ yields g_m

$Im(Z_c) = -\dfrac{2p}{\omega C_s}$ yields f or p

Ref. Vittoz, JSSC June 88, 774-783

Complex plane for 3-pt oscillator : Design crit.



$$Im_0 \approx -\frac{1}{\omega\left(C_3 + \dfrac{C_1C_2}{C_1+C_2}\right)}$$

$$Im_A = -\frac{2p}{\omega_s C_s} \approx Im_0$$

Small p :

Large $C_{1,2}$

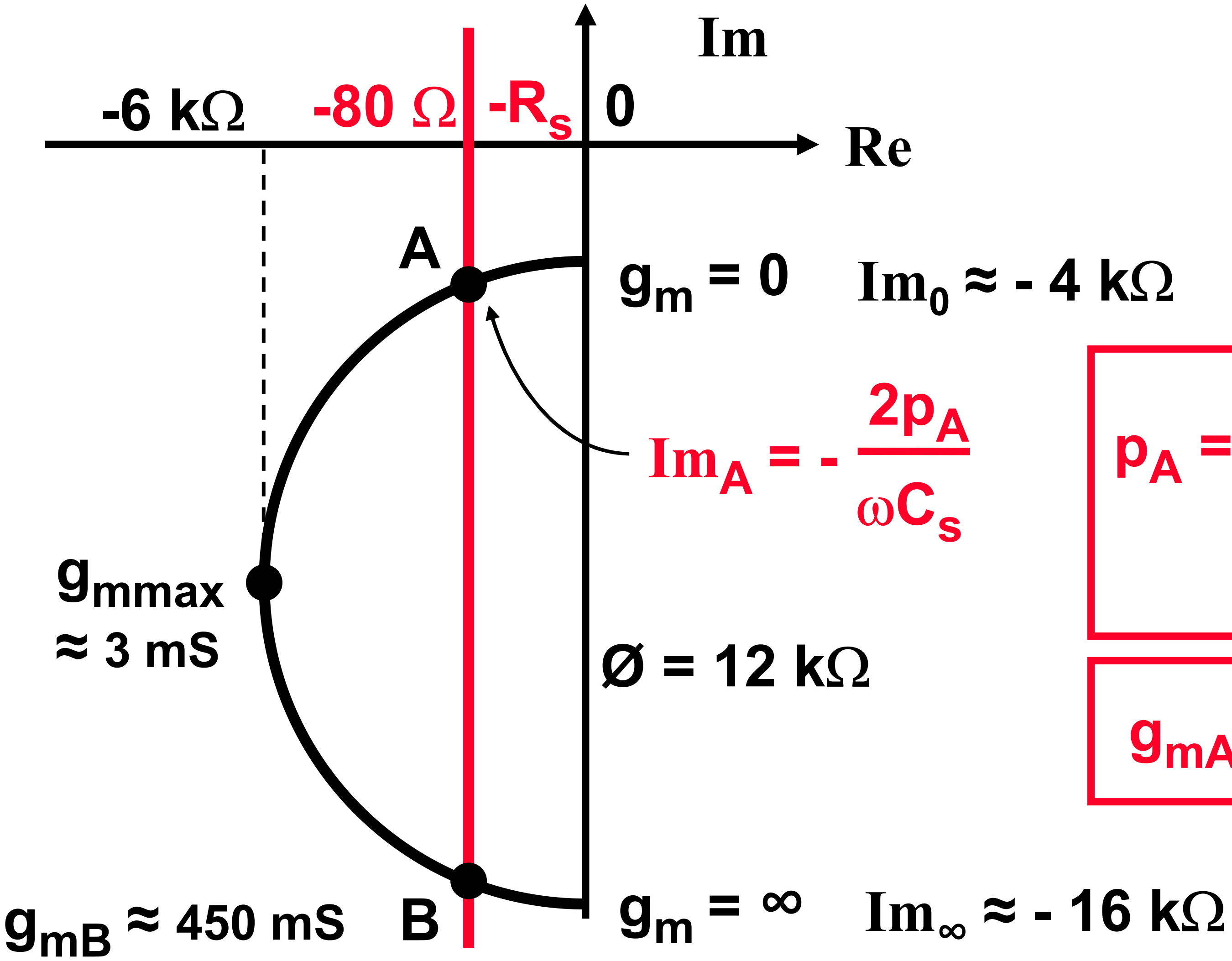
$$\varnothing = \frac{1}{\omega C_3}\,\frac{1}{1 + C_3\dfrac{C_1+C_2}{C_1C_2}}$$

$$Im_\infty \approx -\frac{1}{\omega C_3}$$

Large circle:

Small C_3

Complex plane for 3-pt oscillator : Example



$C_1 = C_2 = 3$ pF
$C_3 = 0.5$ pF
20 MHz   80 Ω

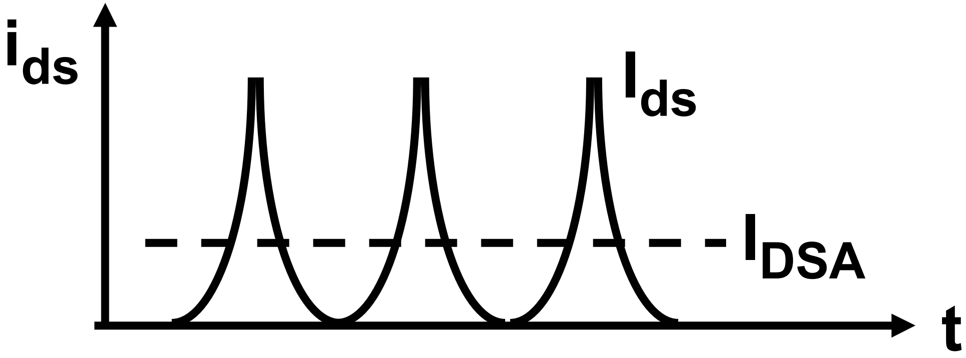
$$Im_A = -\frac{2p_A}{\omega C_s}$$

$$p_A = \frac{C_s}{2\left(C_3 + \frac{C_1 C_2}{C_1 + C_2}\right)}$$

$$g_{mA} \approx R_s C_1 C_2 \omega_s^2$$

≈ 11 μS

Amplitude of oscillation



$$V_{gs} = \frac{I_{ds}}{g_{mA}} = \frac{I_{ds}}{I_{DSA}} \frac{I_{DSA}}{g_{mA}}$$

$\frac{I_{ds}}{I_{DSA}}$ Nonlinear (Bessel) More spiked for higher $C_{1,2}$!!!

$$\frac{2}{\pi} \ldots 2 \quad \frac{V_{GS} - V_T}{2}$$

$$V_{gs} \approx V_{GS} - V_T \quad \text{or} \quad 2n\frac{kT}{q} \quad \text{in wi}$$

Large !

Start-up of oscillation

τ_{min} occurs at $g_m \approx g_{mmax}$

$$\tau_{min} = \frac{L_s}{\mathrm{Re}\,(Z_s) + R_s}$$

Re (Z_s) is half circle Ø

$$\mathrm{Re}\,(Z_s) = \frac{1}{2}\,\frac{1}{\omega_s C_3} \quad \text{if } C_3 << C_1$$

R_s << Re (Z_s)

$$\tau_{min} \approx \frac{2\,C_3}{\omega_s C_s} \approx \frac{400}{\omega_s} \quad \text{since } C_3 \approx 200\,C_s$$

or also $\tau_{min} \approx 2Q\,R_s\,C_3$

Power dissipation

In MOST : $g_{mA} \approx \omega_s^2 R_s C_1 C_2 \approx R_s (C_1 \omega_s)^2$

$$I_{DSA} \approx g_{mA} \frac{V_{GS} - V_T}{2} \approx 2\ \mu A \Longrightarrow 6\ \mu W$$

In X-tal : $I_c = \frac{V_{gs}}{Z_{C1}} = |V_{gs}|\ C_1 \omega_s \approx |V_{GS} - V_T|\ C_1 \omega_s$

$$P_c = \frac{R_s I_c^2}{2} = \frac{R_s}{2} |V_{GS} - V_T|^2 (C_1 \omega_s)^2$$

$$= |V_{GS} - V_T|^2 \frac{g_{mA}}{2} \qquad \approx 0.2\ \mu W$$

Design procedure for X-tal oscillators - 1

X-tal : f_s f_p R_s C_p (or f_s Q C_s C_p) ($Q = 1/\omega_s C_s R_s$)

1. Take : $C_3 > C_p$ but as small as possible

Pulling factor $p = \frac{1}{2} \frac{C_s}{C_3 + \frac{C_1 C_2}{C_1 + C_2}} \approx \frac{1}{2} \frac{C_s}{C_L}$ $\quad C_L = \frac{C_1}{2} = \frac{C_2}{2}$

If $p < \frac{C_s}{4C_p}$ it is a series oscillator (best !)

If $p >$ it is a parallel oscillator (not stable !)

Choose C_L large (> C_3), subject to power dissipation !

Design procedure for X-tal oscillators - 2

2. Calculate $g_{mA} \approx R_s C_L^2 \omega_s^2$ $\left(\approx \dfrac{\omega_s}{C_s Q} C_L C_L\right)$

and take $g_{mStart} \approx 10\ g_{mA}$

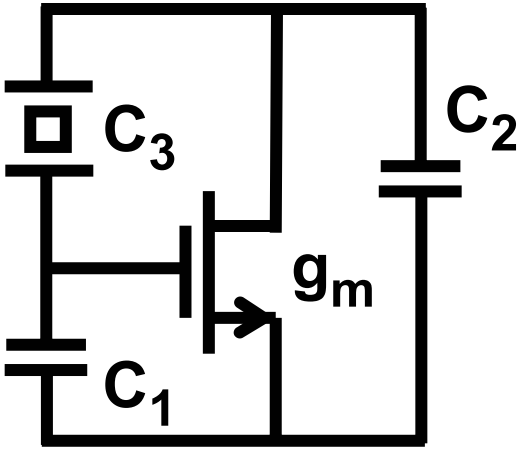
3. Choose $V_{GS} - V_T$, which gives the amplitude V_{gs}

and current $I_{DS} = \dfrac{g_m (V_{GS} - V_T)}{2}$ and $\dfrac{W}{L}$

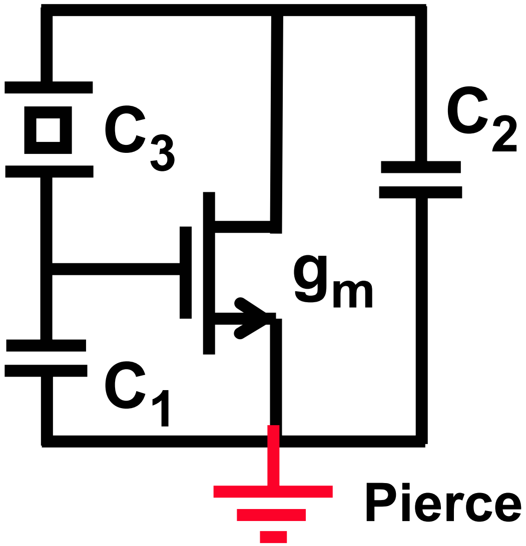
and power $P = (V_{GS} - V_T)^2 \dfrac{g_m}{2}$

4. Verify that biasing $R_B > 1/(R_s C_3^2 \omega_s^2)$

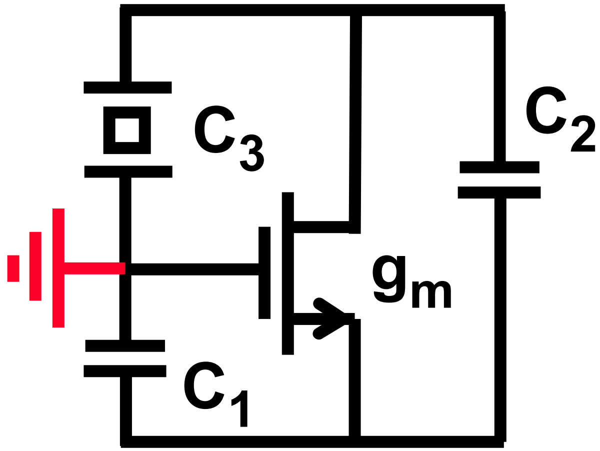
Single-transistor X-tal oscillator



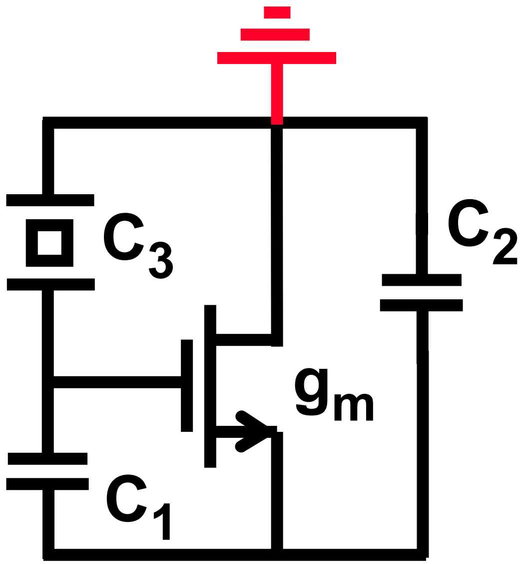
Basic three-point oscillator



Pierce

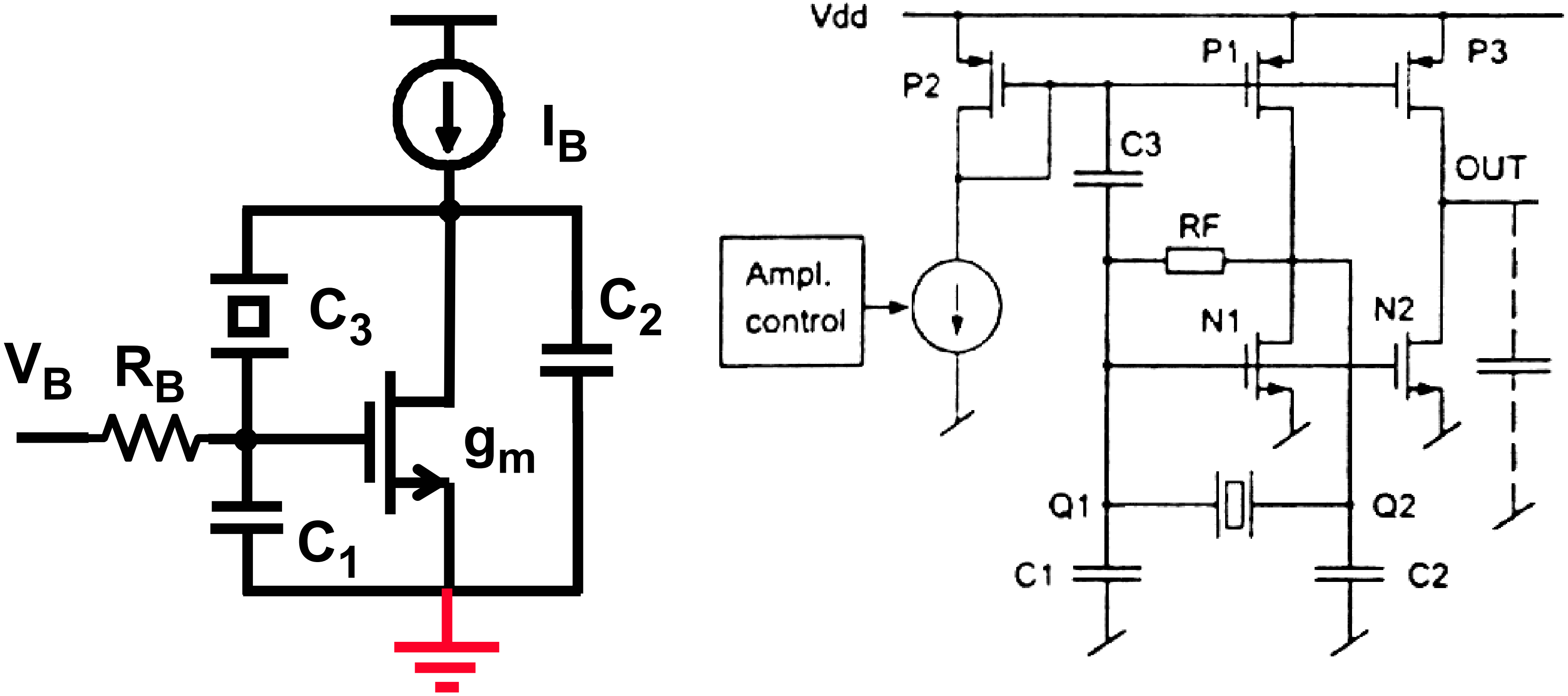


Colpitts : 1-pin X=D



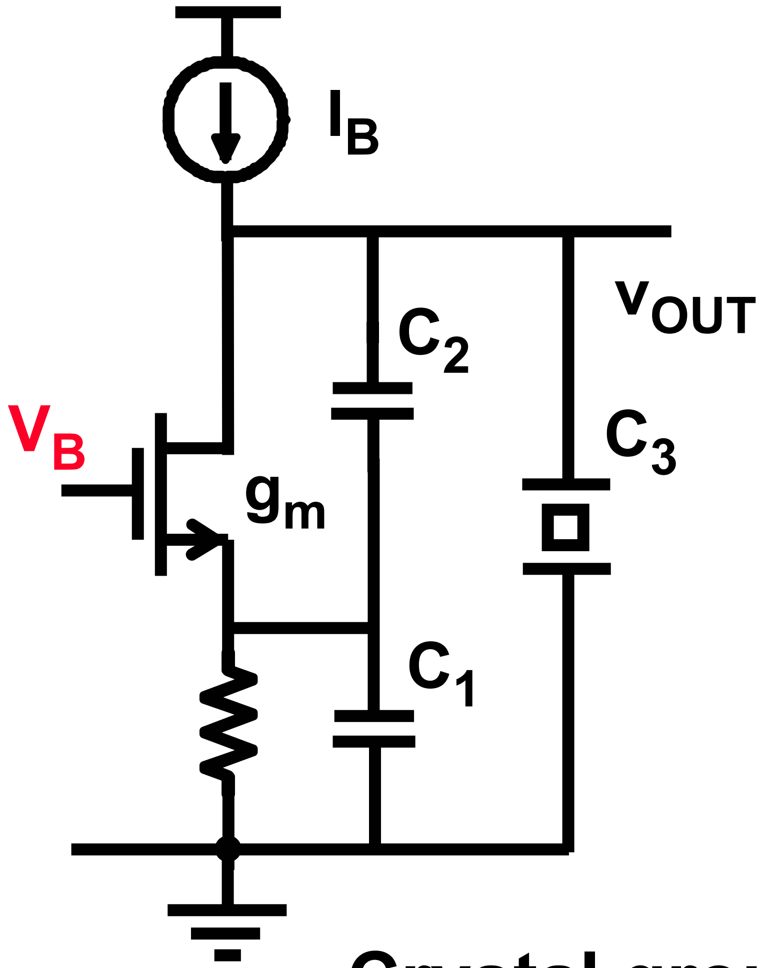
Santos : 1-pin X=G

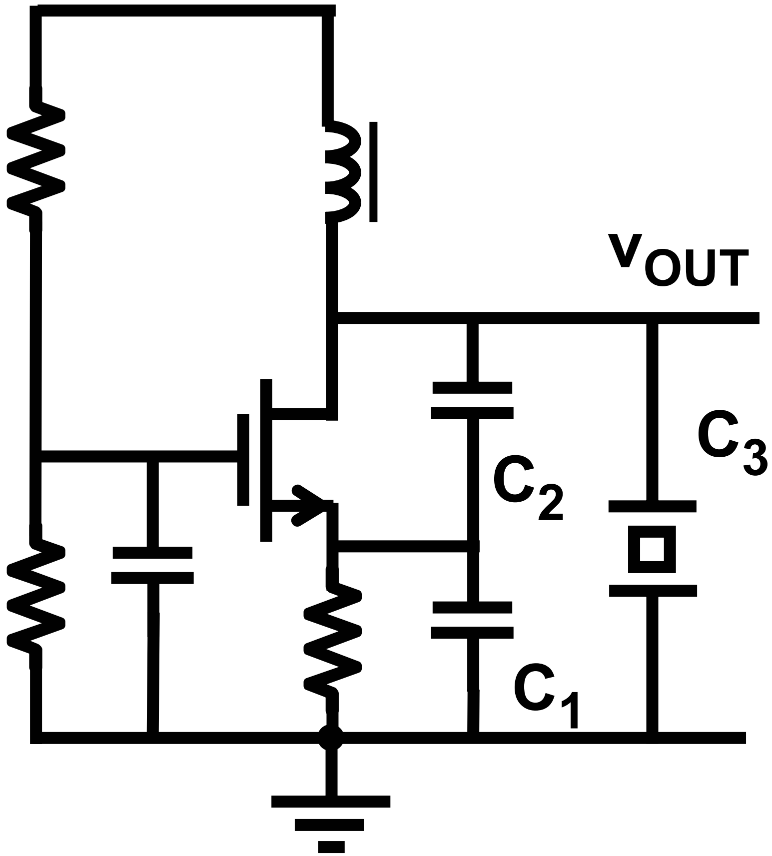
Pierce X-tal oscillator



32 kHz 1.2 V 78 nA

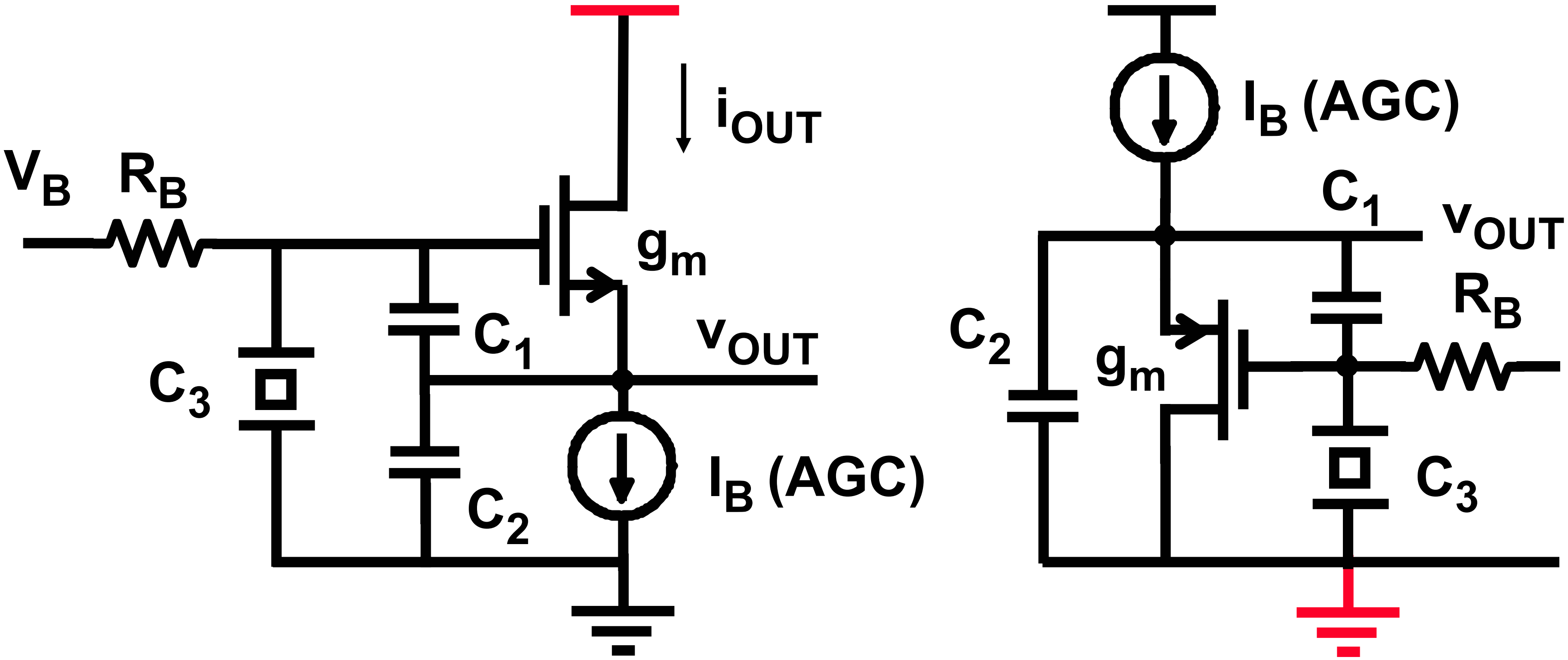
Colpitts X-tal oscillator





Crystal grounded : single-pin : X = D

Santos X-tal oscillator



Crystal grounded : single-pin : X = G

Ref. Santos, JSSC April 84, 228-236   Ref. Redman-White, JSSC Feb.90, 282-288

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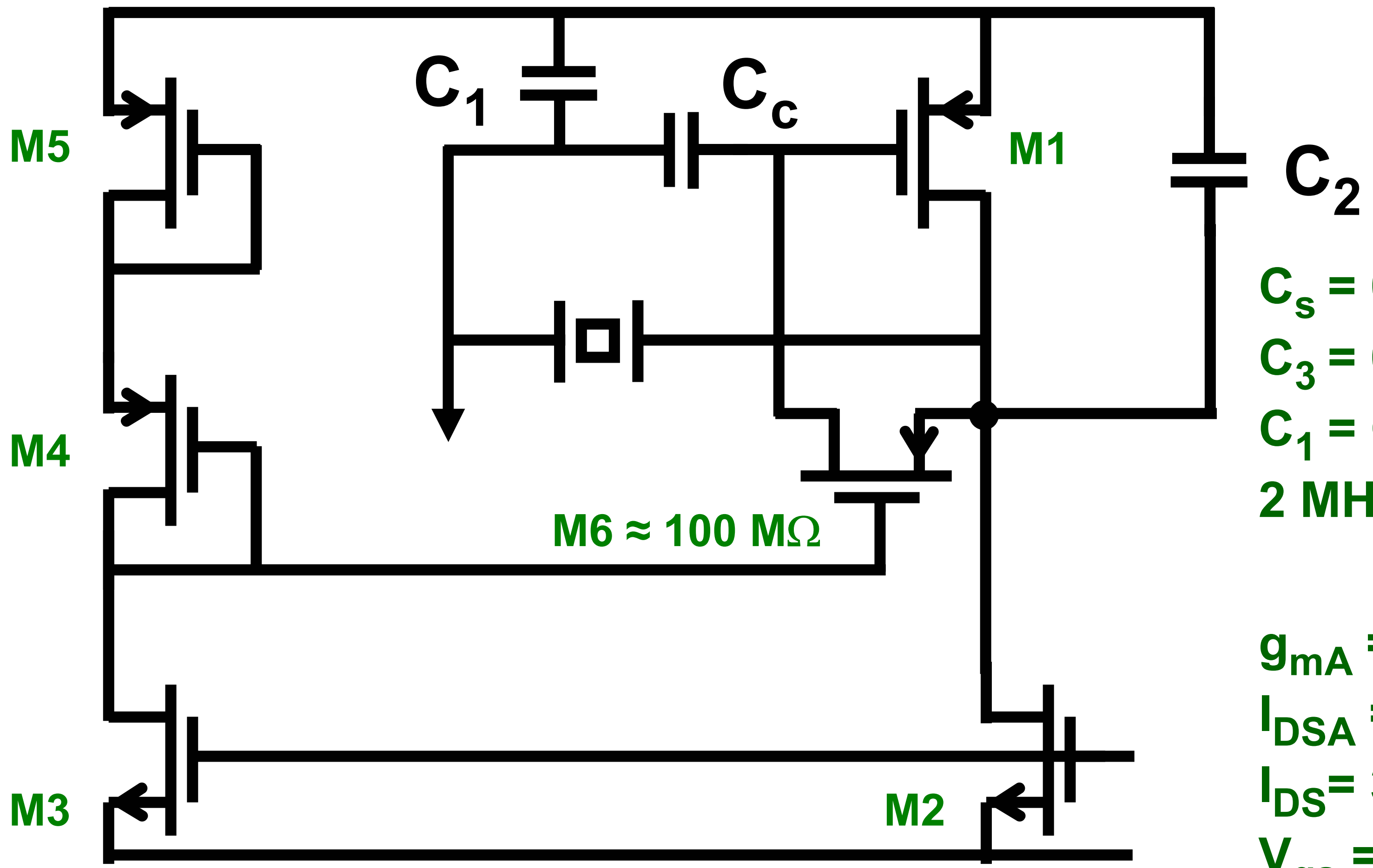
Practical Pierce X-tal oscillator

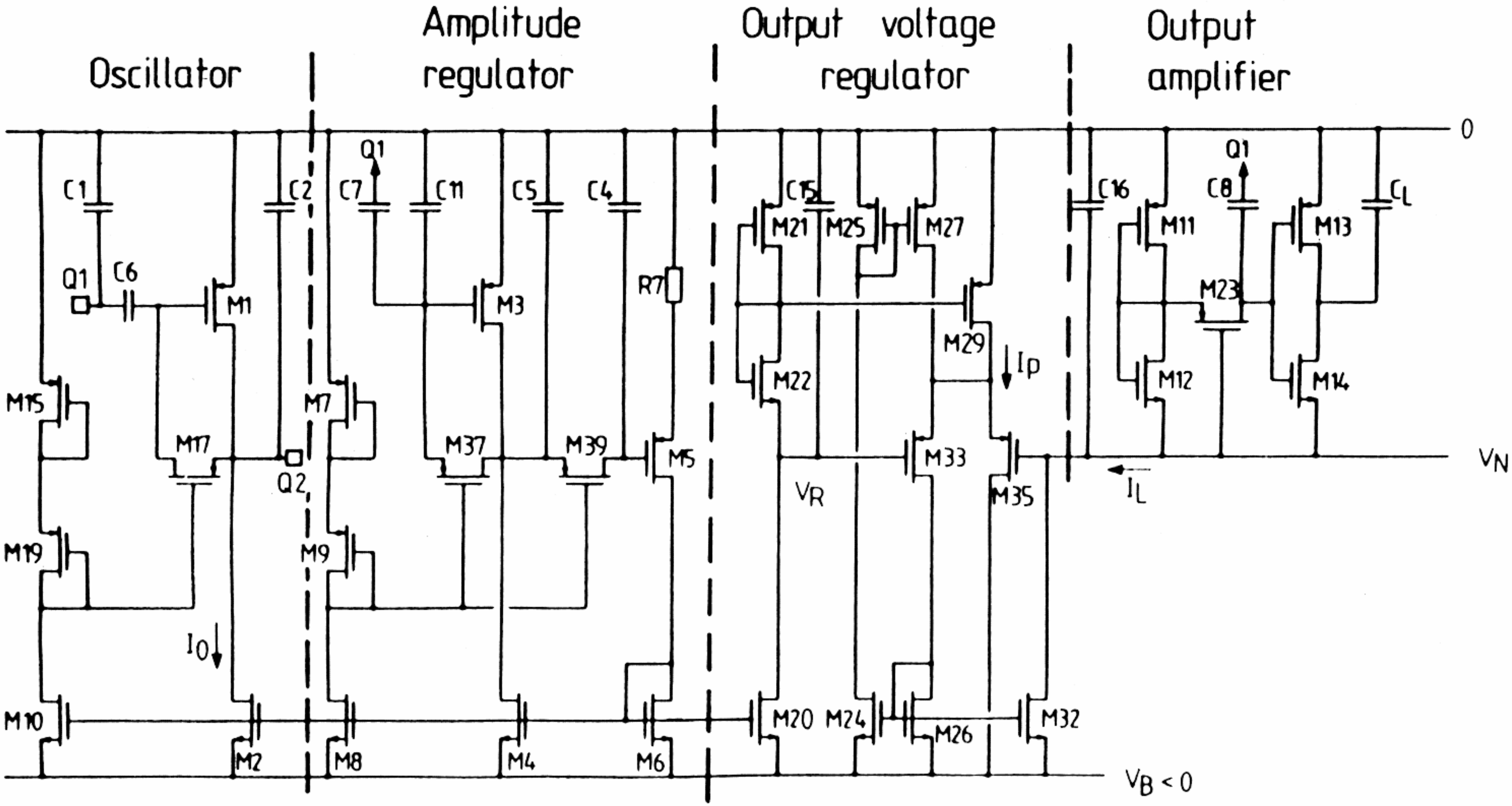


$C_s = 0.5$ fF
$C_3 = 0.6$ pF
$C_1 = C_2 = 2.8$ pF
2 MHz

$g_{mA} = 2\ \mu S$
$I_{DSA} = 80$ nA
$I_{DS} = 350$ nA
$V_{gs} = 300$ mV

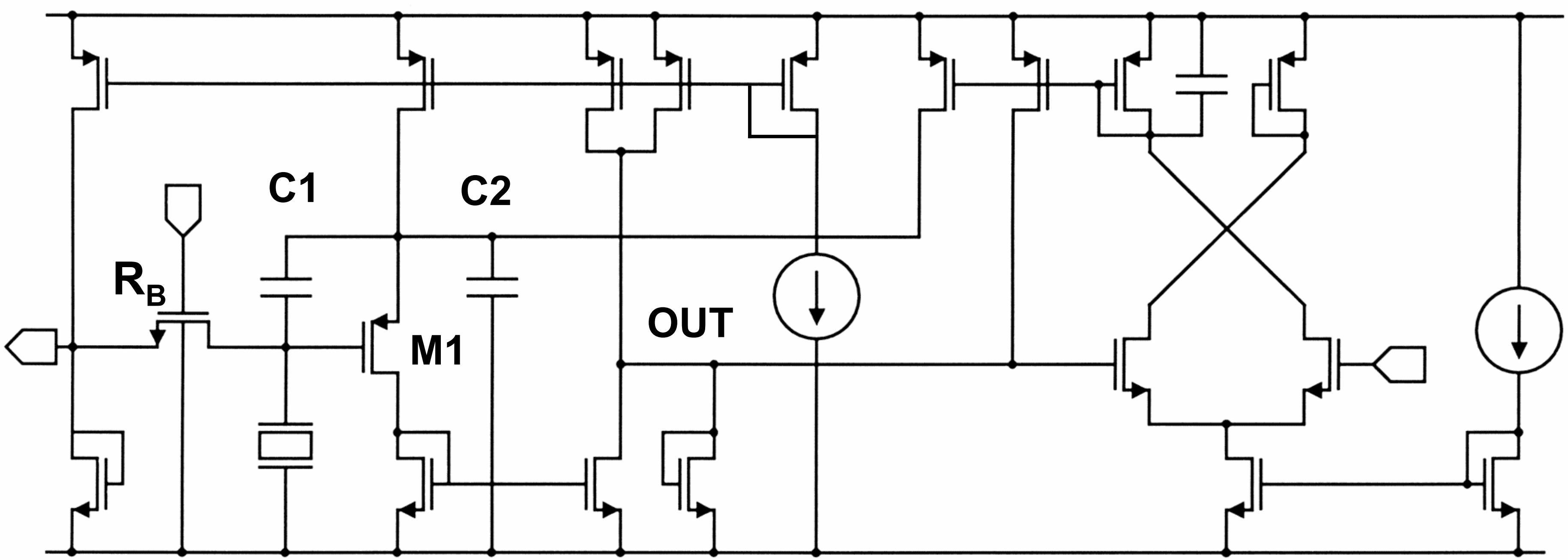
Ref. Vittoz, JSSC June 88, 774-783

Full schematic



Ref. Vittoz, JSSC June 88, 774-783

Single-pin oscillator with crystal to Gate

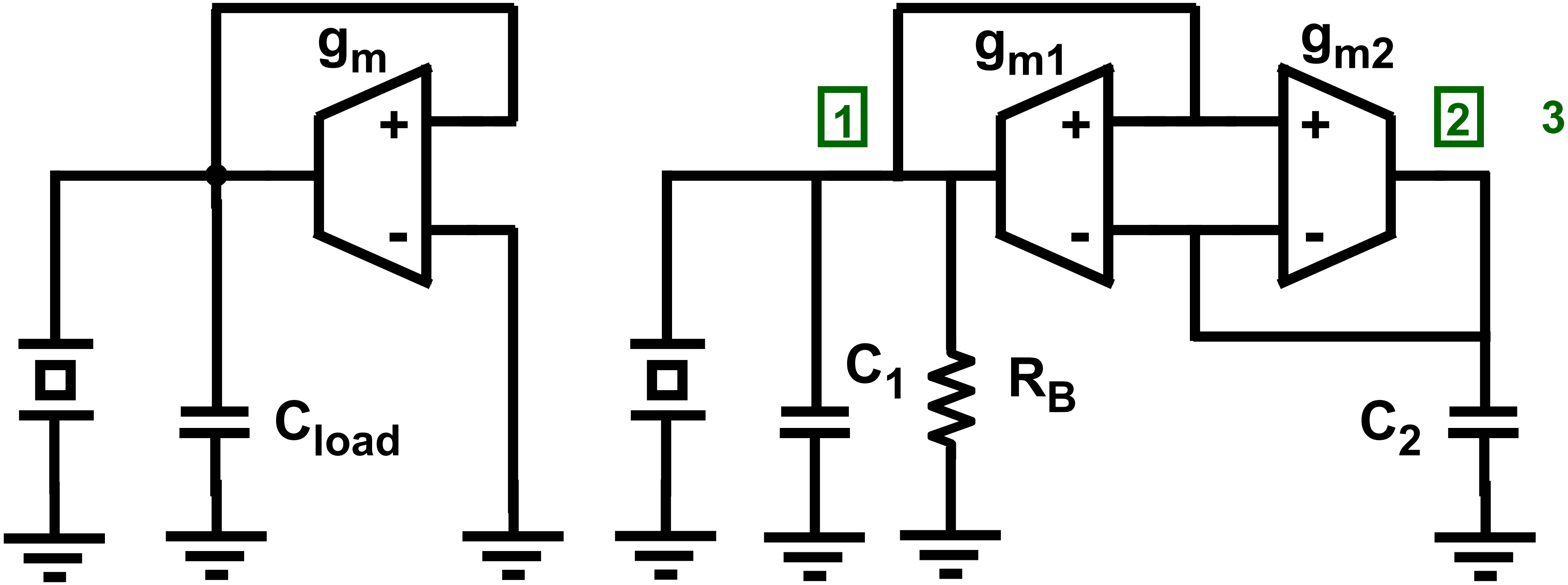


f_s = 9.9956 MHz
f_p = 10.012 MHz

C_s = 24.3 fF
C_o = 7.4 pF
L = 10.4 mH
R = 7.2 Ω (?)

p = 0.8 10^{-3}
$C_1 = C_2$ = 50 pF
g_{mA} = 350 μS
I_{DSA} = 90 μA ($V_{GS}-V_T$ = 0.5 V)

Single-pin oscillator - 1



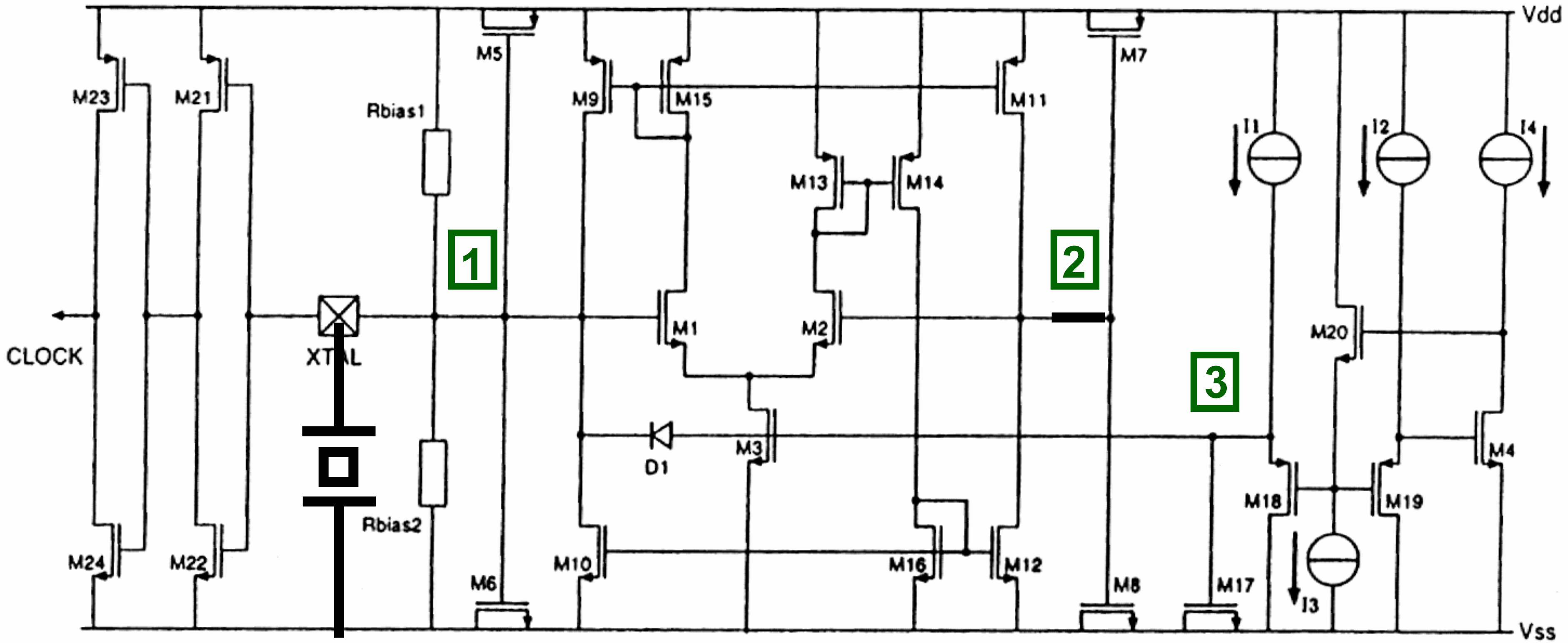
$g_m = R_s (C_s \omega_0)^2$

DC unstable !

Positive FB dominant at crystal frequency !

Ref. van den Homberg, JSSC July 99, 956-961

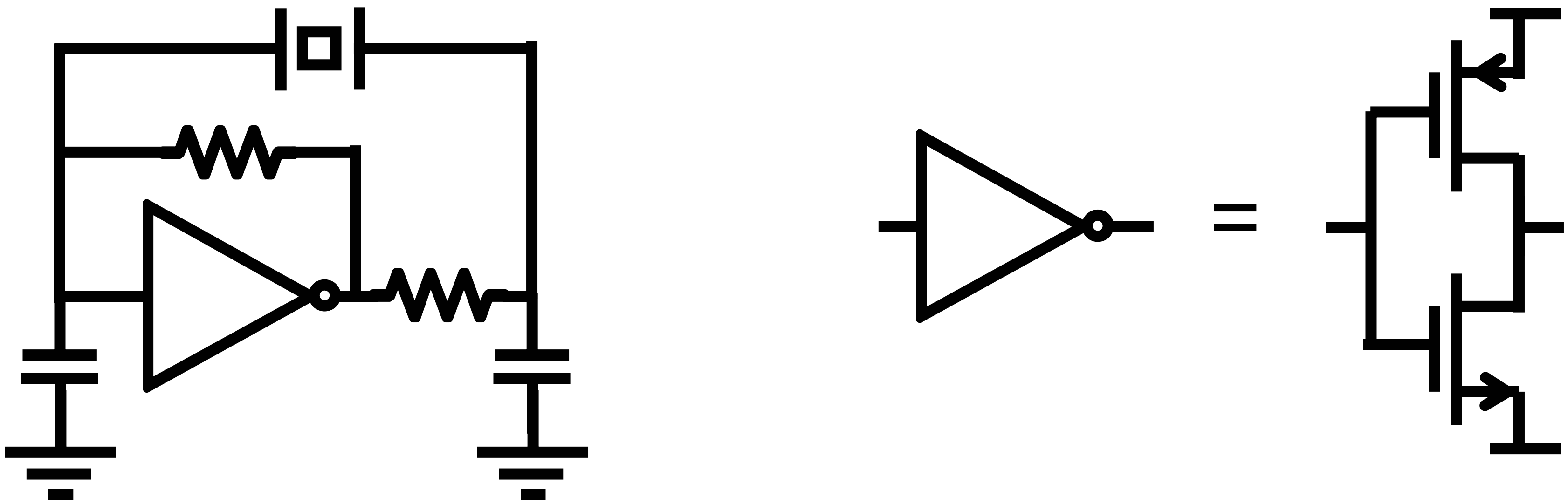
Single-pin oscillator - 2

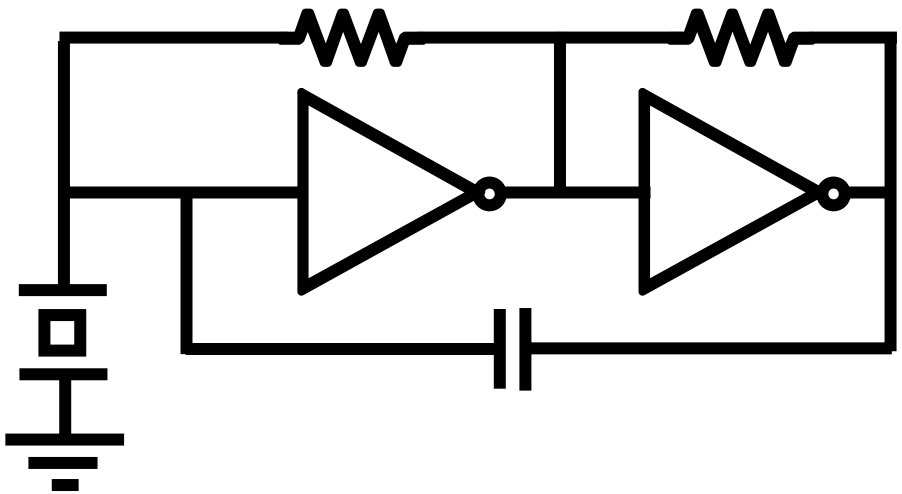


10 MHz, 3.3 V, 0.35 mA

Ref. van den Homberg, JSSC July 99, 956-961

X-tal oscillators with CMOS inverters





Large current peaks !
Bad PSRR !!

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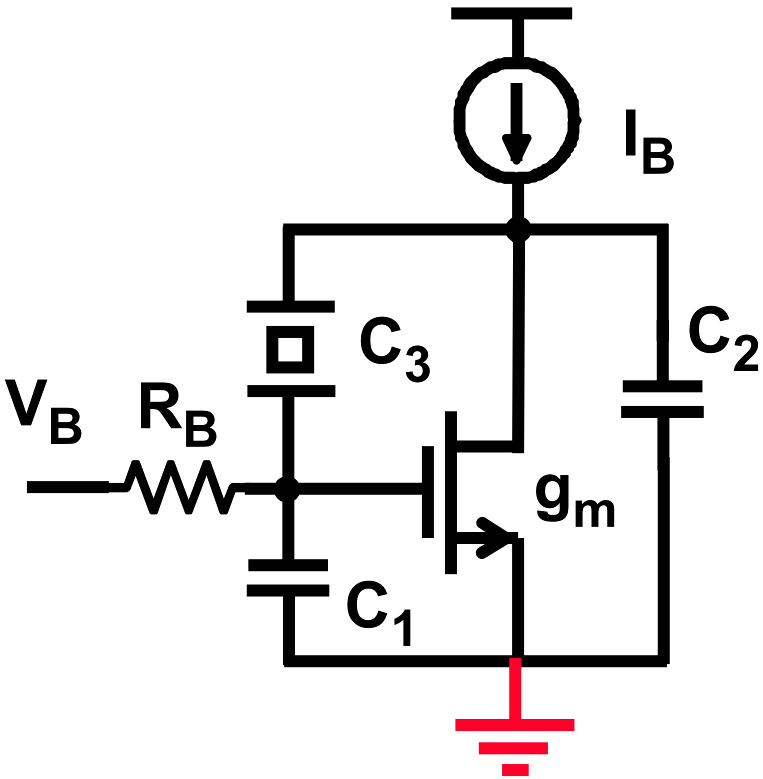
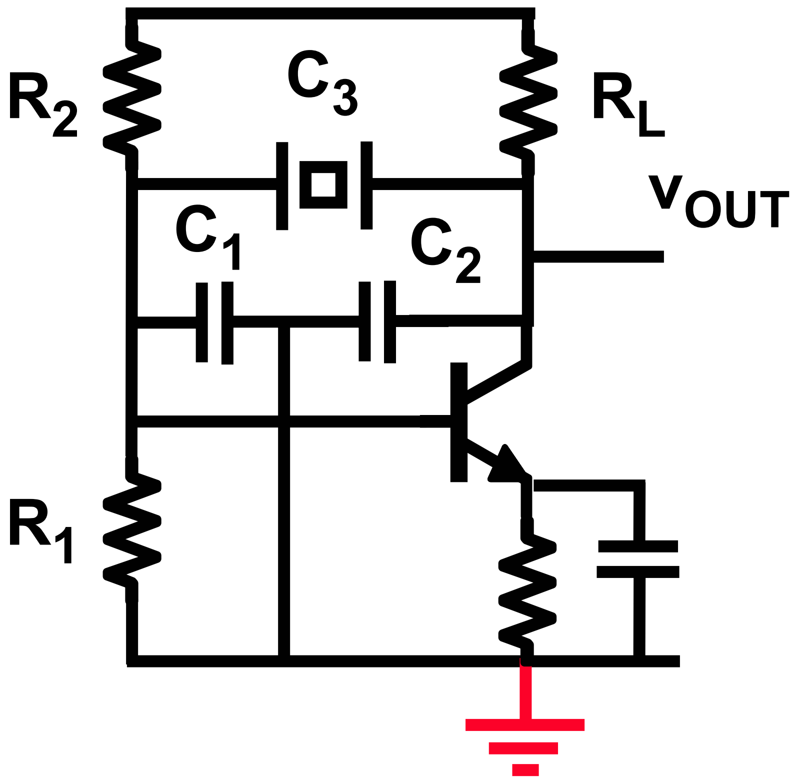
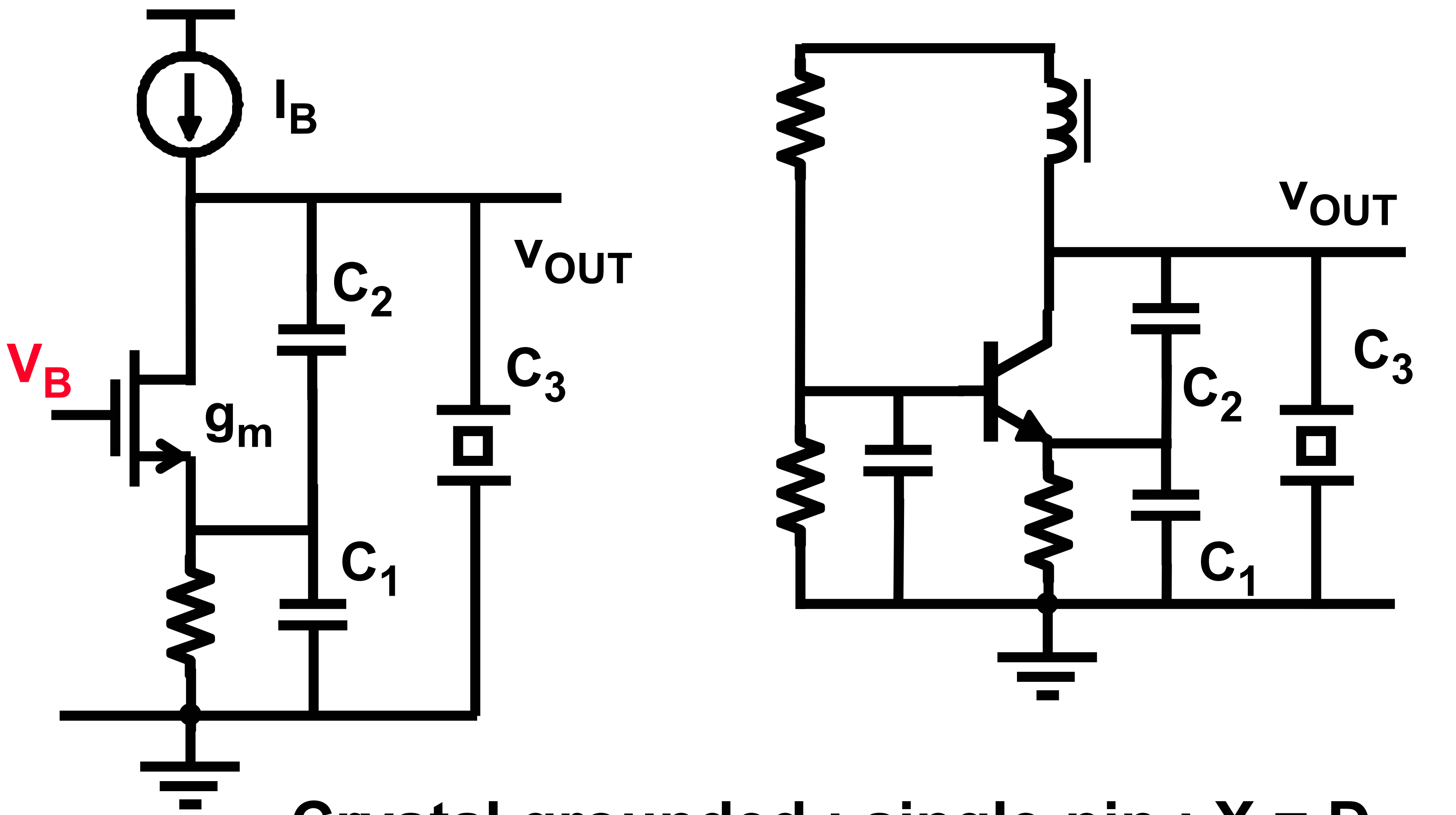
Pierce X-tal oscillator



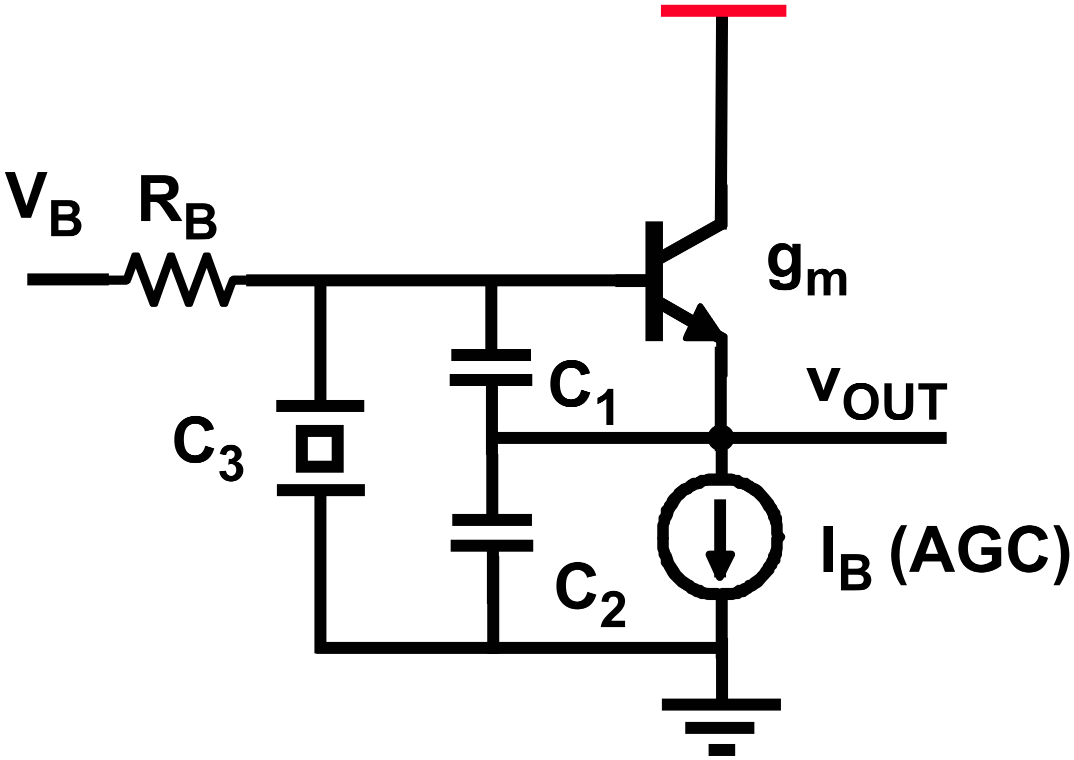


Colpitts X-tal oscillator



Crystal grounded : single-pin : X = D

Santos X-tal oscillator



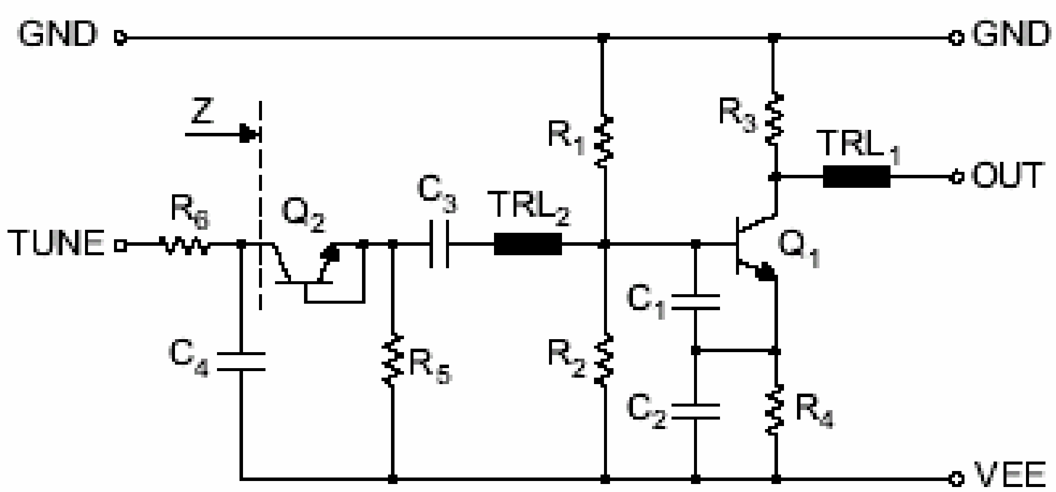
Crystal grounded : single-pin : X = G

Buffered output

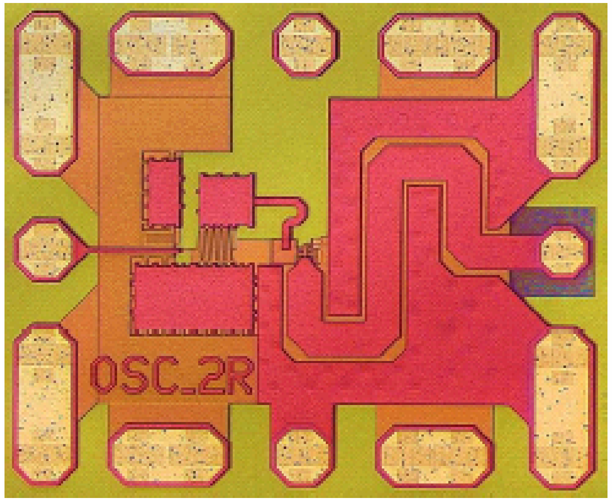
Ref. Santos, JSSC April 84, 228-236

Ref. Redman-White, JSSC Feb.90, 282-288

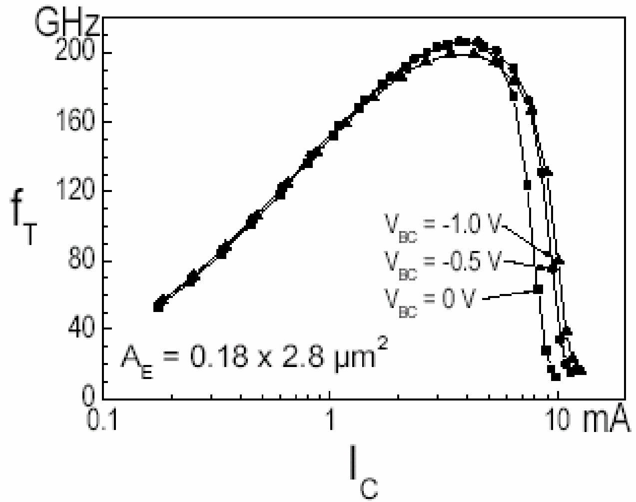
98 GHz VCO in SiGe Bipolar technology



Colpitts



0.55 x 0.45 mm

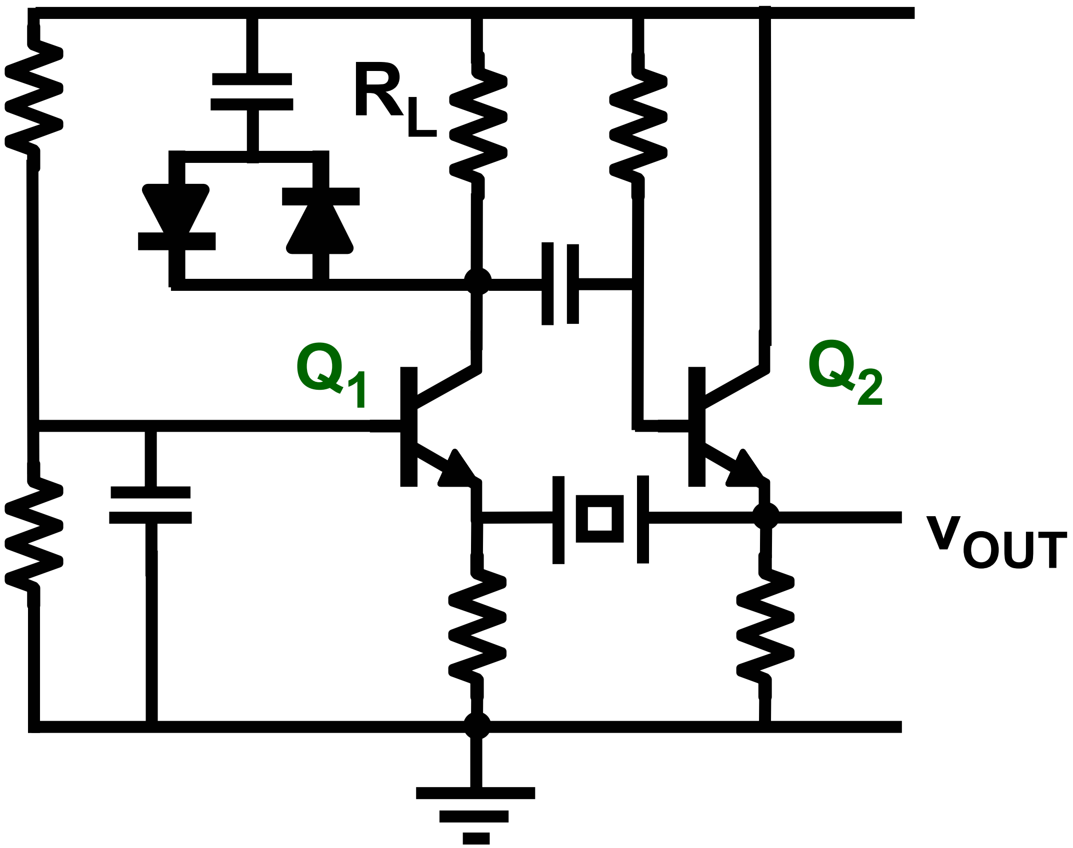


12 mA at - 5 V

-97 dBc/Hz at 1 MHz

Ref. Prendl BCTM Toulouse 03

Positive feedback circuits - 1

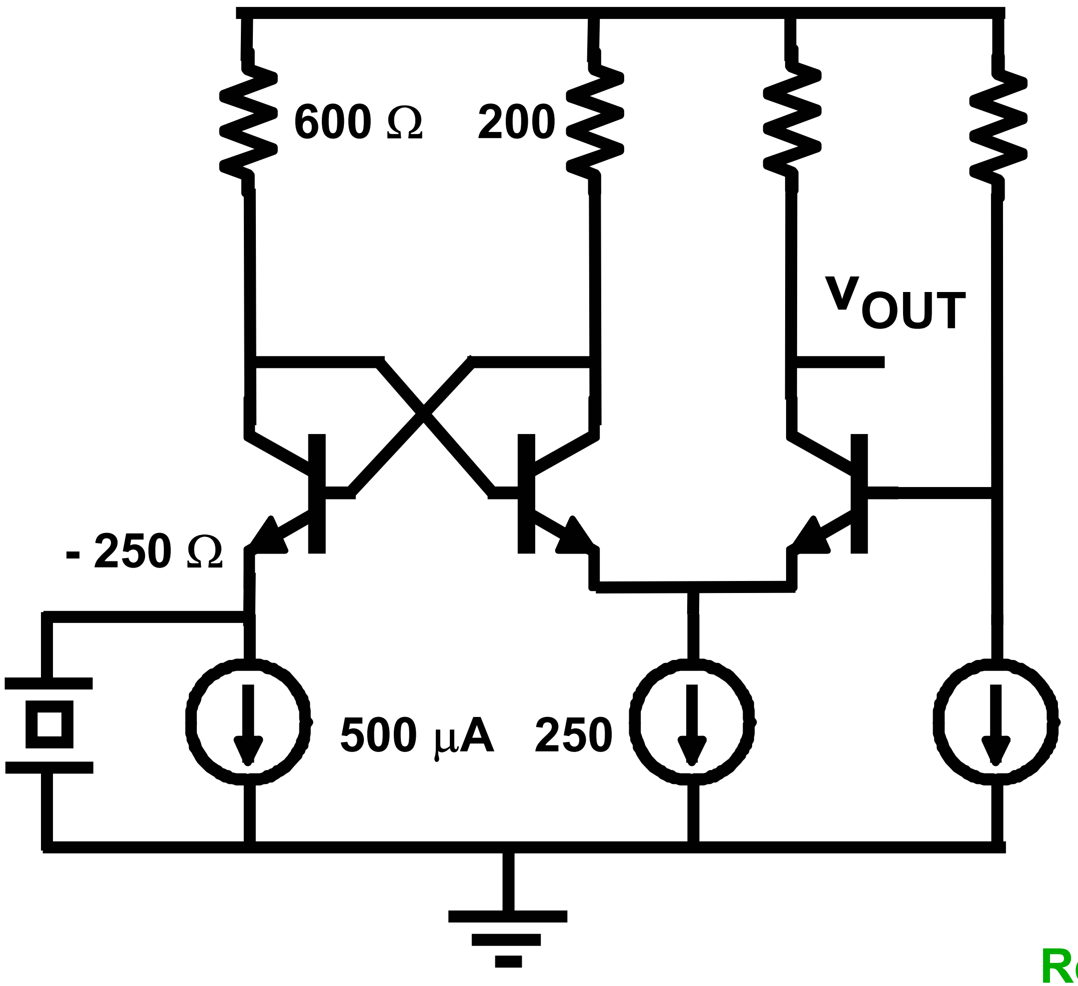


$T = g_{m1} R_L$

$R_L > R_s$

Ref. Nordholt, CAS 90, 175-182

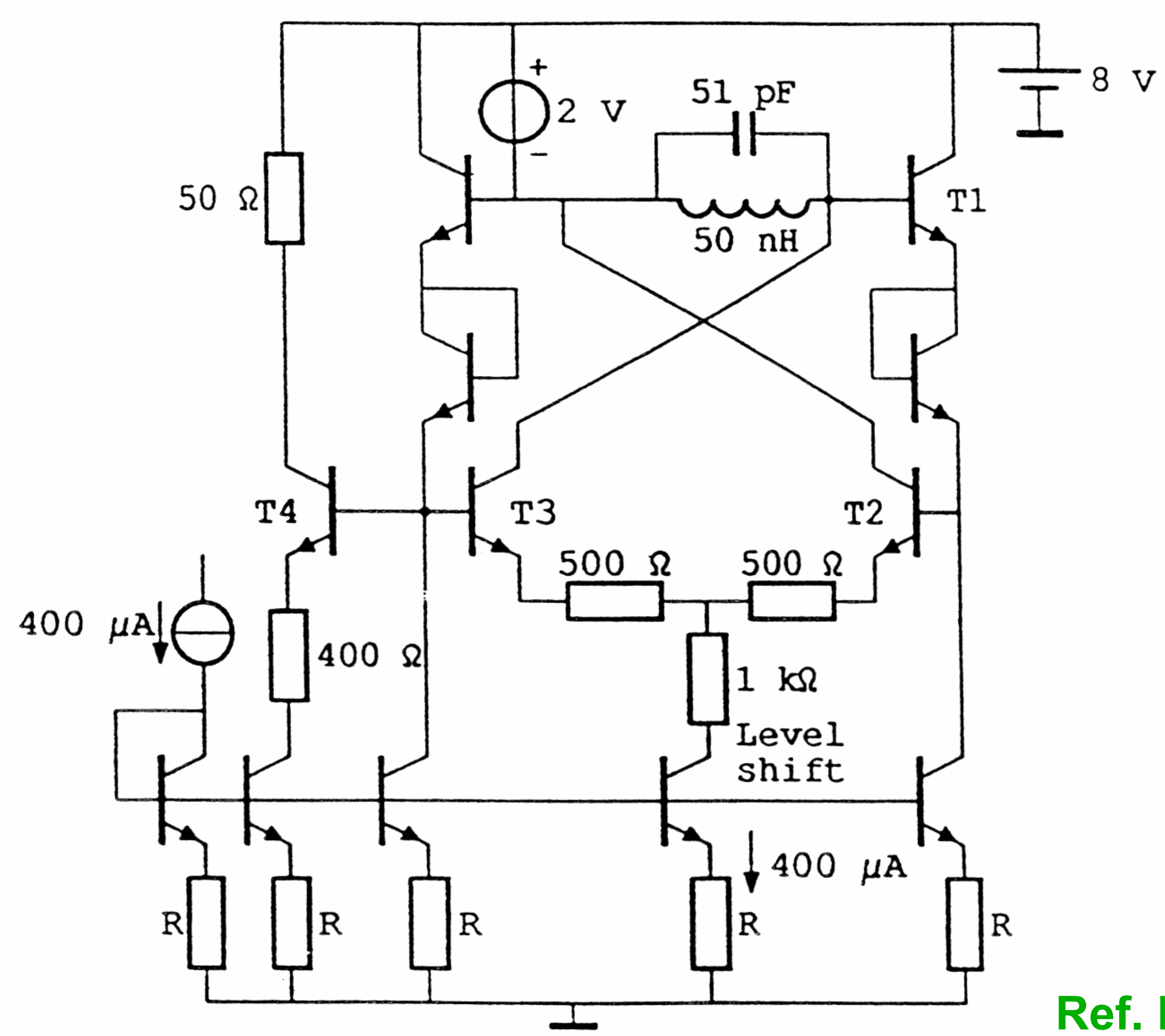
Positive feedback circuits - 2



Buffered ouput !

Ref. Nordholt, CAS 90, 175-182

Positive feedback circuits - 3



g_{mA} = 8 mS

100 MHz

Ref. Nordholt, CAS 90, 175-182

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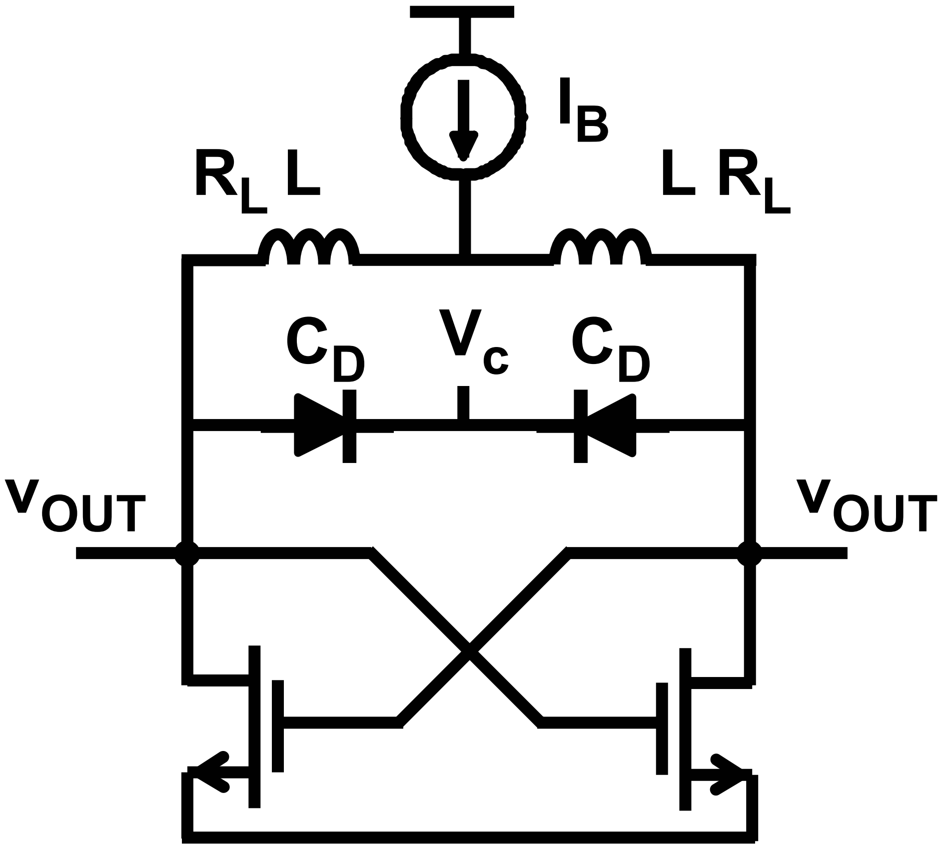
Voltage Controlled Oscillator



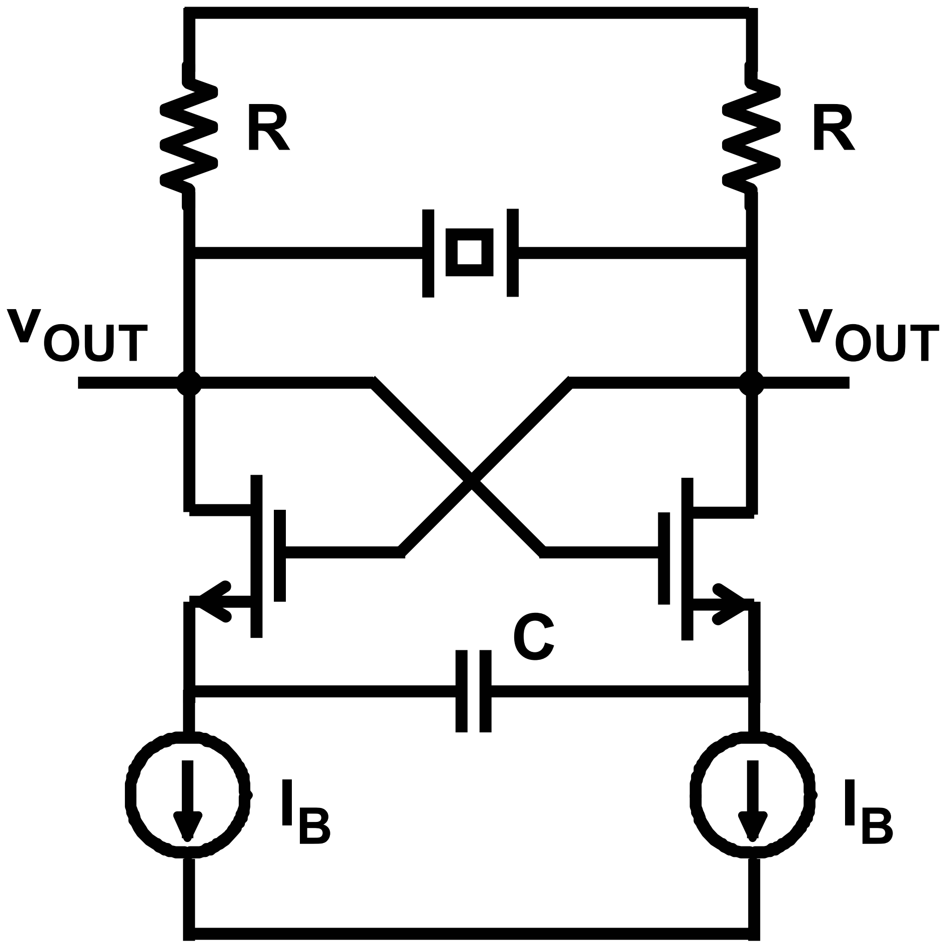
$$\omega_s = \frac{1}{\sqrt{LC_D}}$$

$$g_{mA} \approx R_L(C_D\omega_s)^2$$

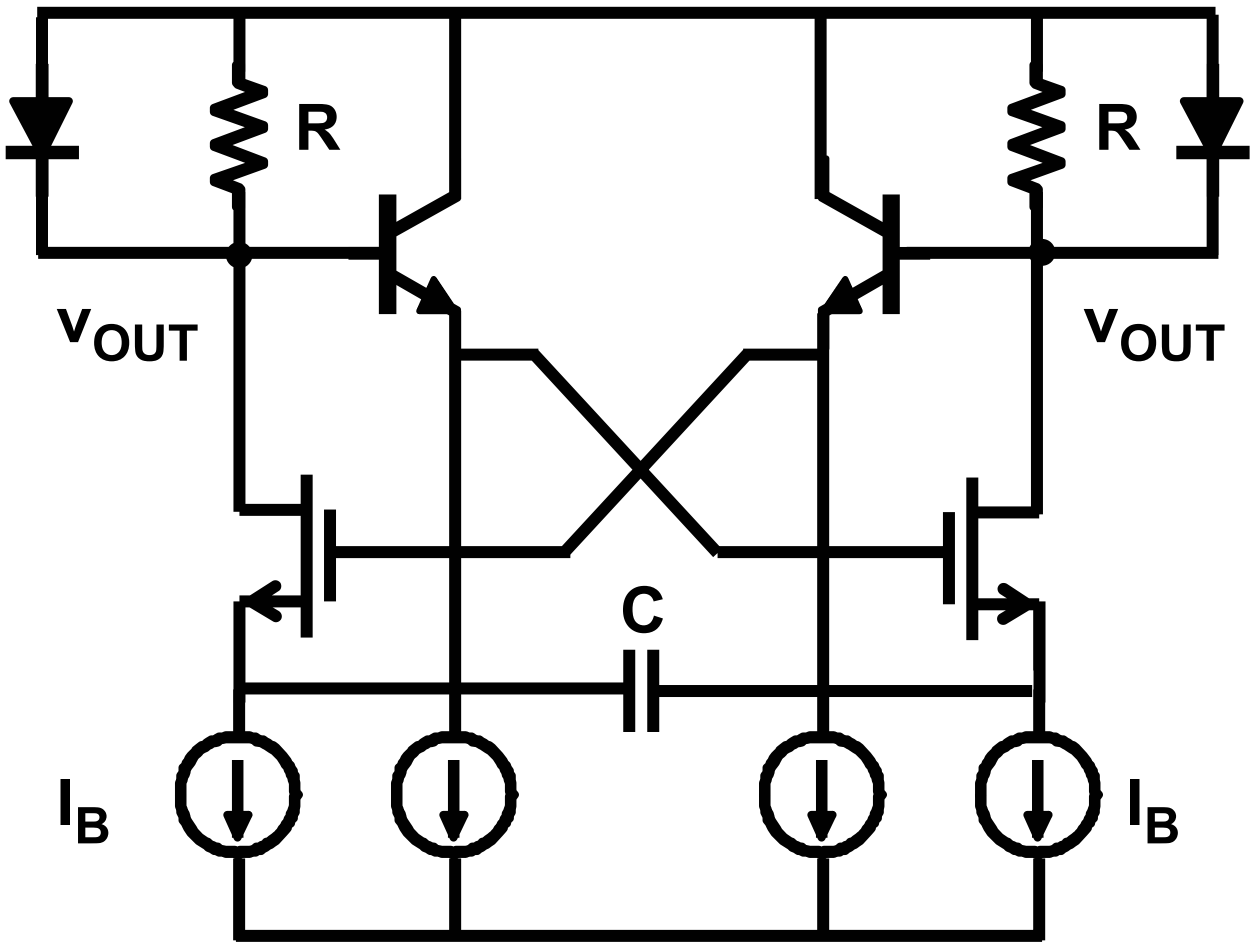
$$\overline{dv_{out}^2}\{\Delta\omega\} = 4kTR_L\left(1 + \frac{4}{3}\right)\left(\frac{\omega_s}{\Delta\omega}\right)^2 df$$

Ref. Craninckx, ACD Kluwer 96, 383-400 ; JSSC May 97, 736-744

Differential crystal Oscillator

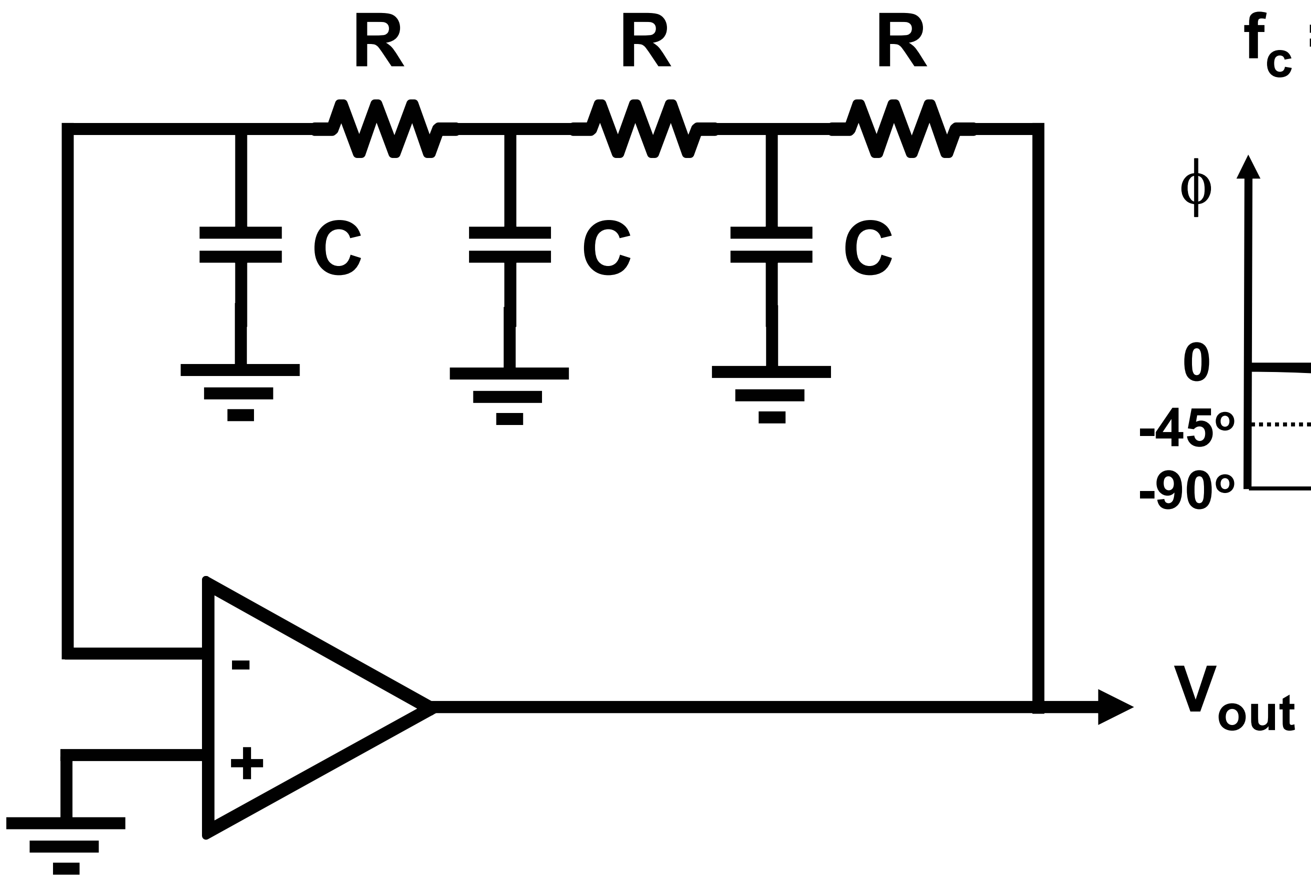


Relaxation Oscillator

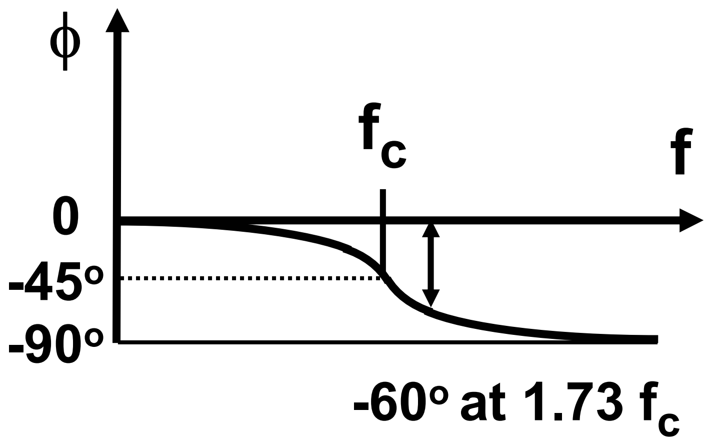


Ref. Grebene, JSSC, Aug.69, 110-122; Gray, Meyer, Wiley, 1984.

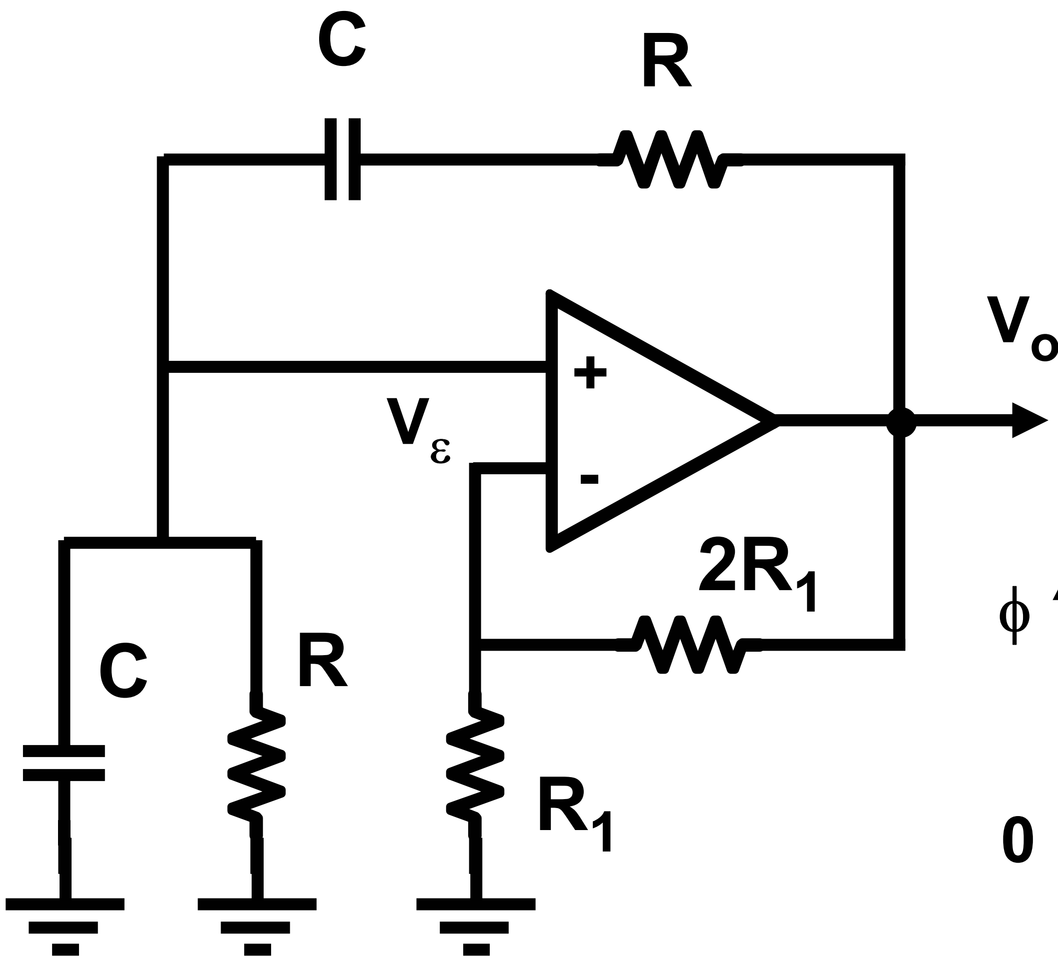
RC Oscillators : 3 x 60° = 180°



$$f_c = \frac{1}{2\pi RC}$$



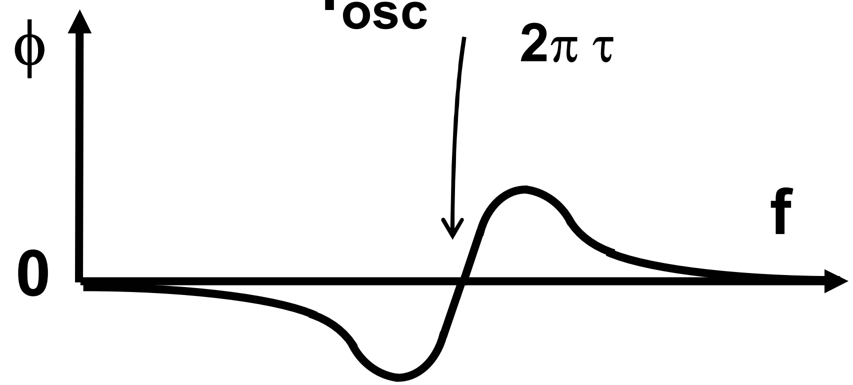
Wien Oscillator : 3 x Gain required



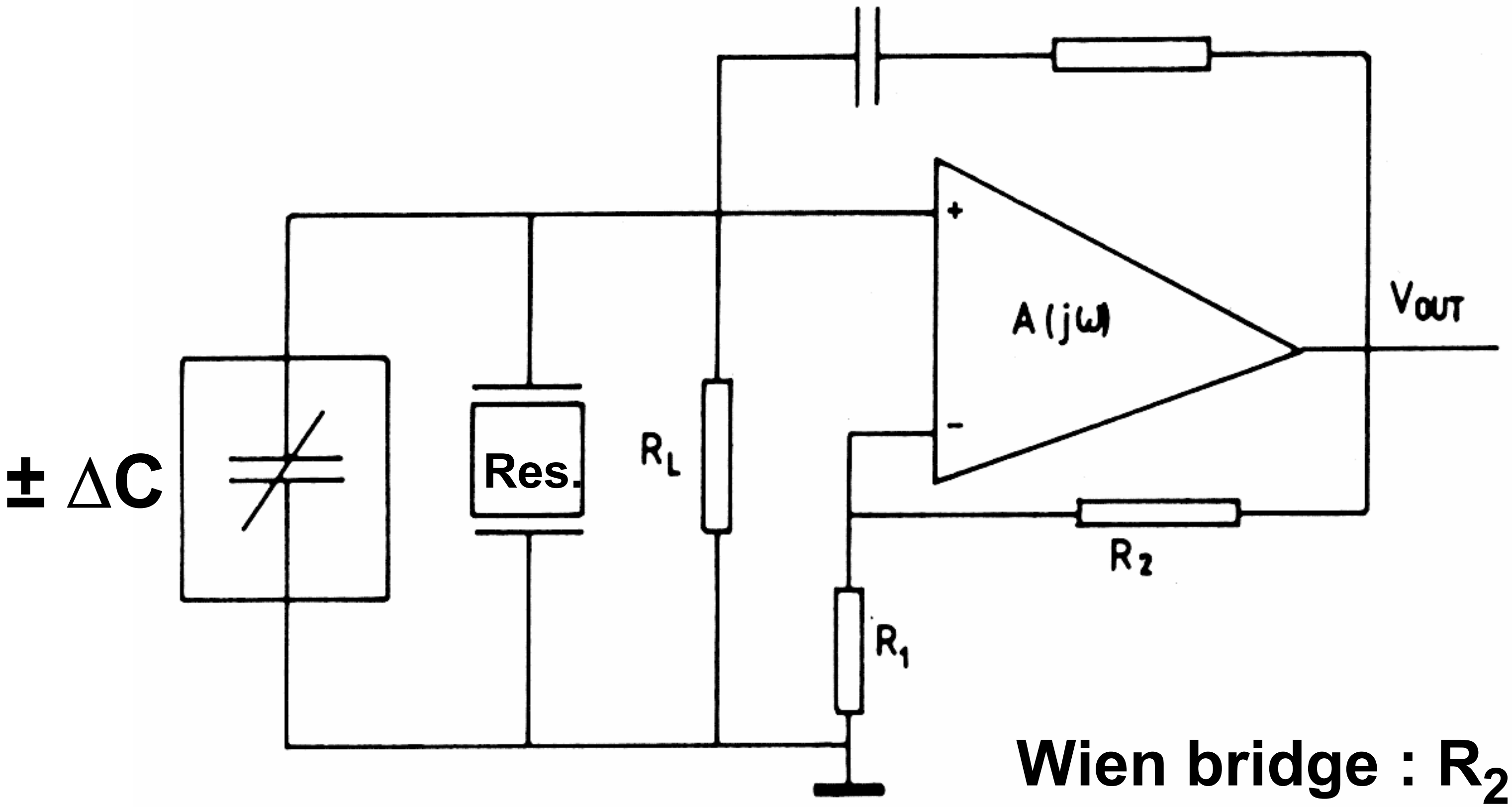
$$\frac{V_\varepsilon}{V_{out}} = \frac{1}{3}\,\frac{1 + 2\tau s + \tau^2 s^2}{1 + 3\tau s + \tau^2 s^2}$$

$$\tau = RC$$

$$f_{osc} = \frac{1}{2\pi\tau}$$



Voltage-controlled X-tal oscillator

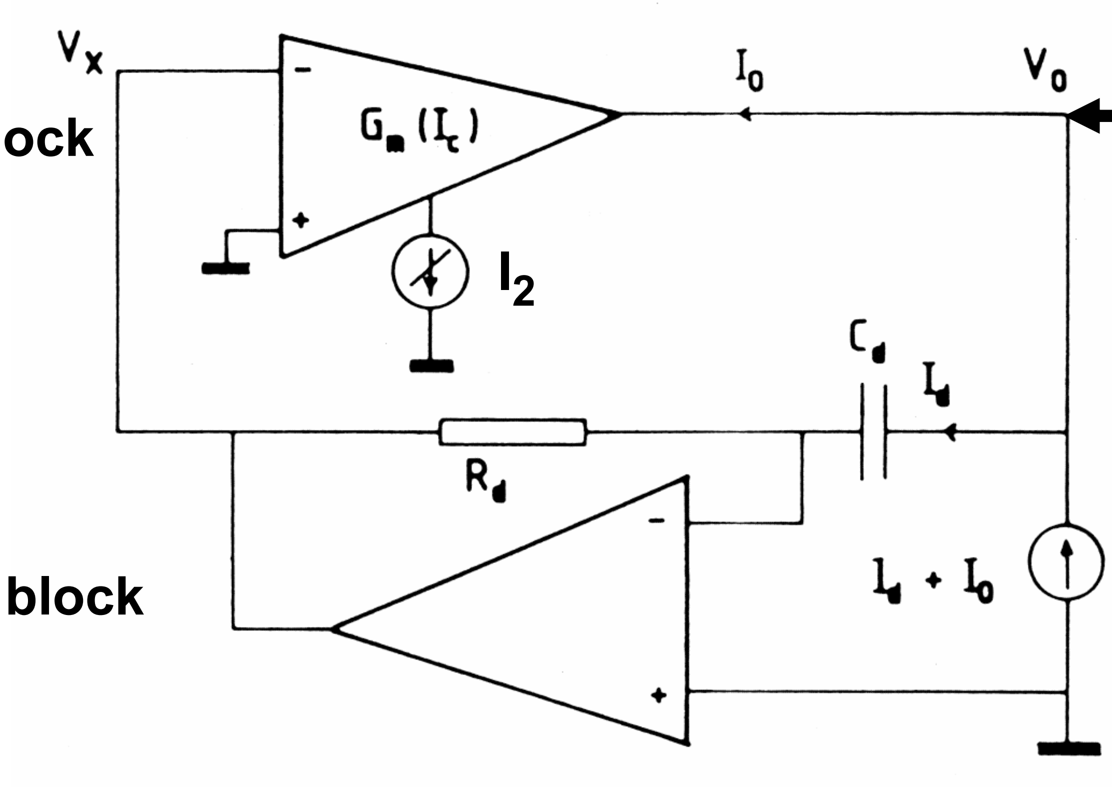


Wien bridge : $R_2 = 2\ R_1$

Resonator 457 kHz
Tuning ± 5 kHz

Ref. Huang, JSSC June 88, 784-793

Variable capacitance ± ΔC



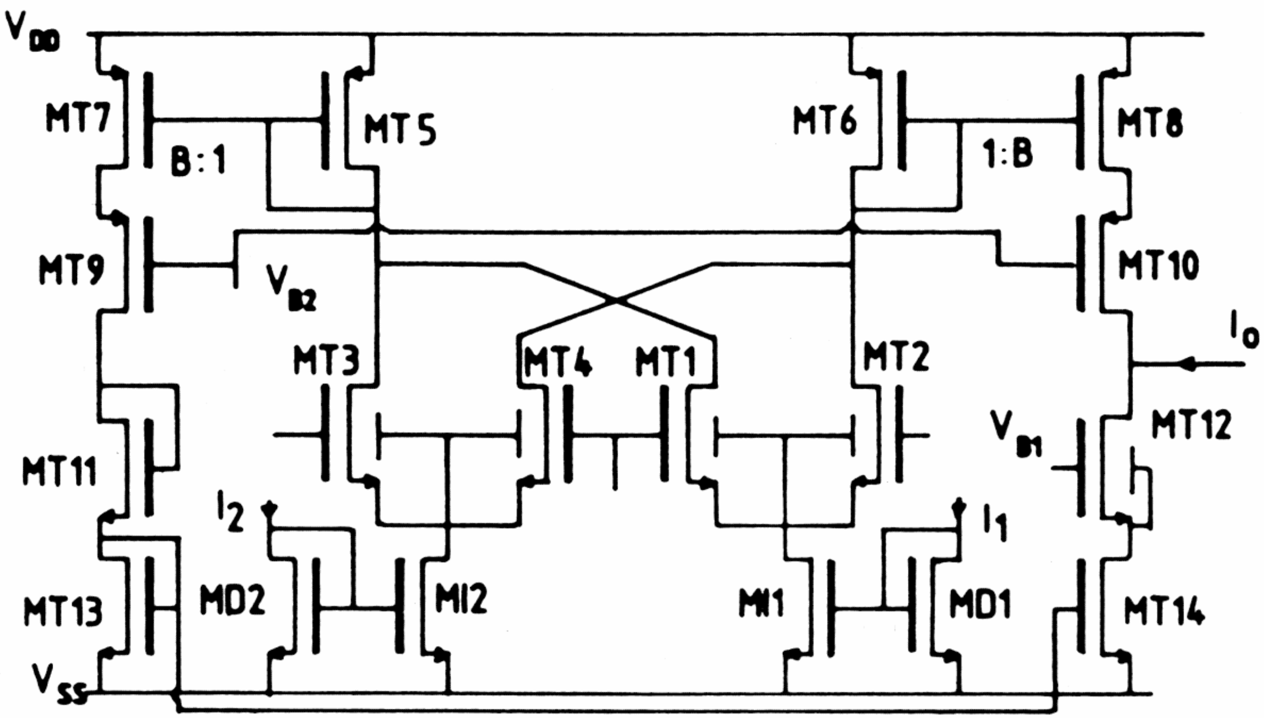
$\pm \Delta C$

$Y_{in} = sC_d \underbrace{G_m R_d}_{25}$

$\pm \Delta G_m$ with I_2

Ref. Huang JSSC June 88, 784-793

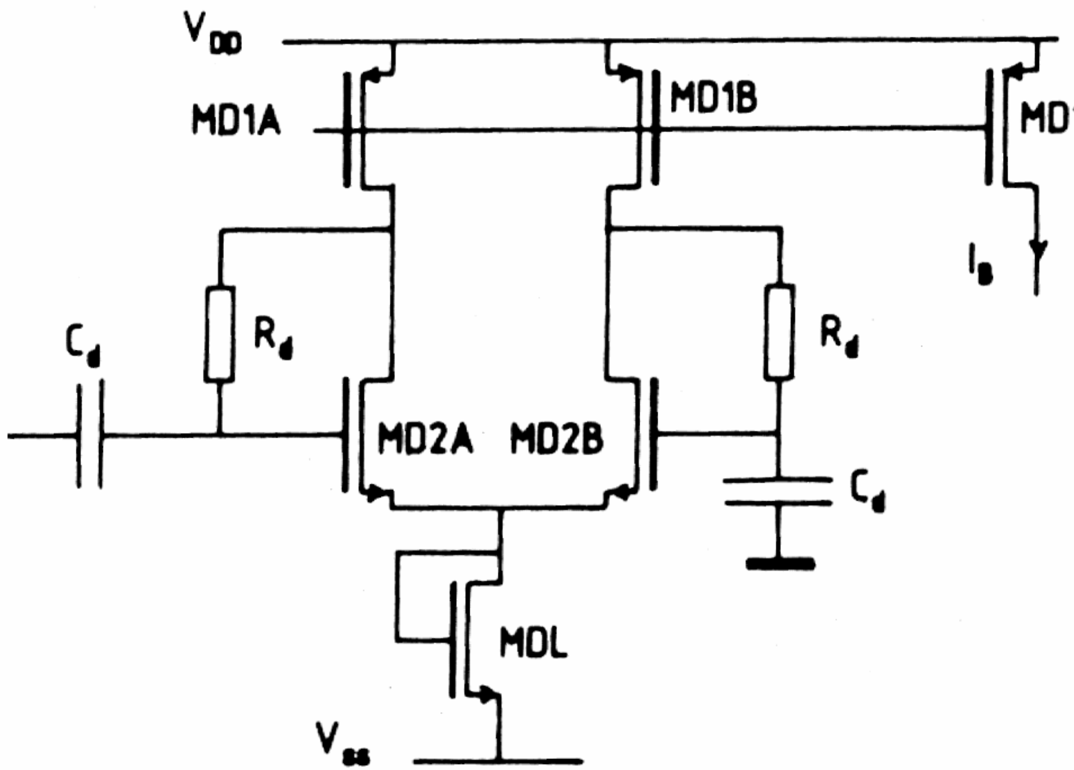
G_m block to generate $\pm \Delta G_m$



$I_1 = 90\ \mu A$

$I_2 = 0 \ldots 180\ \mu A$ $\quad G_m = B\,[(2\beta I_1)^{1/2} - (2\beta I_2)^{1/2}]$

R_dC_d block as differentiator



C_d = 4 pF

R_d = 40 kΩ

Ref. Huang JSSC June 88, 784-793

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References X-tal oscillators -1

A.Abidi, "Low-noise oscillators, PLL's and synthesizers", in R. van de Plassche, W.Sansen, H. Huijsing, "Analog Circuit Design", Kluwer Academic Publishers, 1997.

J. Craninckx, M. Steyaert, "Low-phase-noise gigahertz voltage-controlled oscillators in CMOS", in H. Huijsing, R. van de Plassche, W.Sansen, "Analog Circuit Design", Kluwer Academic Publishers, 1996, pp. 383-400.

Q.T. Huang, W. Sansen, M. Steyaert, P.Van Peteghem, "Design and implementation of a CMOS VCXO for FM stereo decoders", IEEE Journal Solid-State Circuits Vol. 23, No.3, June 1988, pp. 784-793.

E. Nordholt, C. Boon, "Single-pin crystal oscillators" IEEE Trans. Circuits. Syst. Vol.37, No.2, Feb.1990, pp.175-182.

D. Pederson, K.Mayaram, "Analog integrated circuits for communications"", Kluwer Academic Publishers, 1991.

References X-tal oscillators - 2

W. Redman-White, R. Dunn, R. Lucas, P. Smithers, "A radiation hard AGC stabilised SOS crystal oscillator", IEEE Journal Solid-State Circuits Vol. 25, No.1, Feb. 1990, pp. 282-288.

J. Santos, R. Meyer, "A one pin crystal oscillator for VLSI circuits", IEEE Journal Solid-State Circuits Vol. 19, No.2, April 1984, pp. 228-236.

M. Soyer, "Design considerations for high-frequency crystal oscillators", IEEE Journal Solid-State Circuits Vol. 26, No.9, June 1991, pp. 889-893.

E. Vittoz, M. Degrauwe, S. Bitz, "High-performance crystal oscillator circuits: Theory and application", IEEE Journal Solid-State Circuits Vol. 23, No.3, June 1988, pp. 774-783.

V. von Kaenel, E. Vittoz, D. Aebischer, " Crystal oscillators", in H. Huijsing, R. van de Plassche, W.Sansen, "Analog Circuit Design", Kluwer Academic Publishers, 1996, pp. 369-382.

Appendix: Polar diagrams

Willy Sansen

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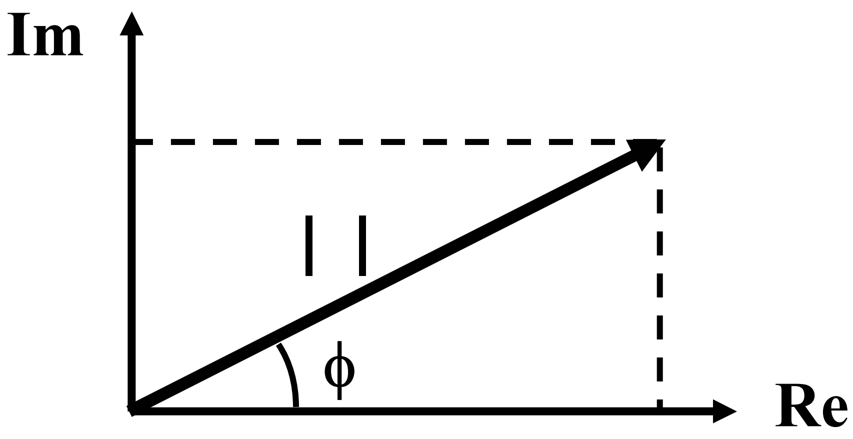
Amplitude, phase, Real & Imaginary



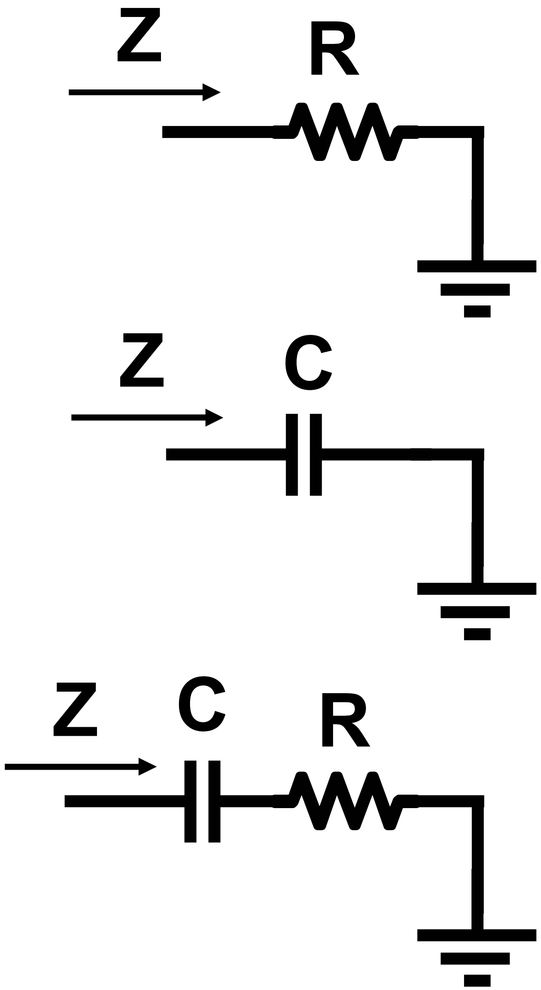
$$| \; | = \sqrt{Re^2 + Im^2}$$

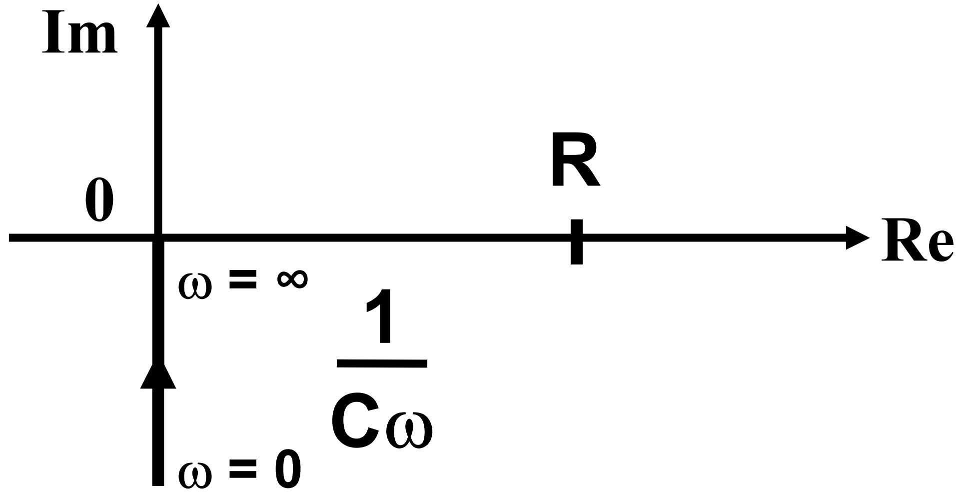
$$tg(\phi) = \frac{Im}{Re}$$

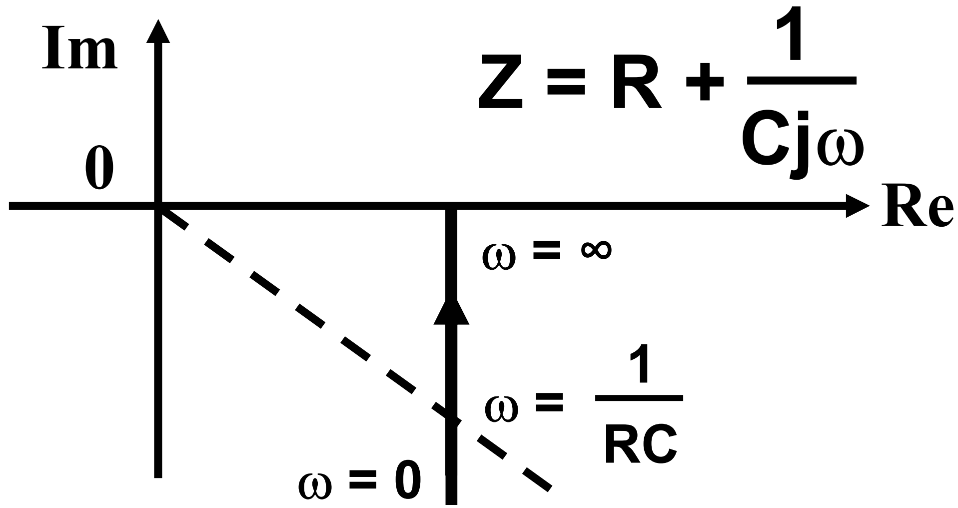
$$Re = | \; | \cos(\phi)$$

$$Im = | \; | \sin(\phi)$$

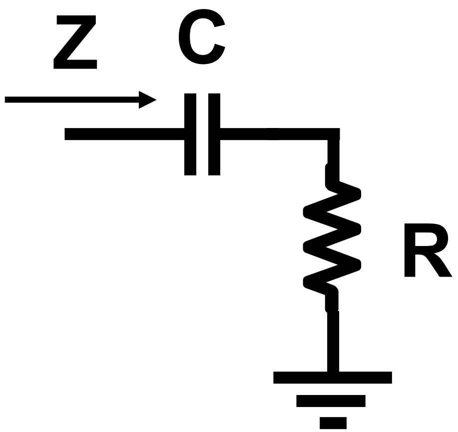
Polar diagram of RC network - 1



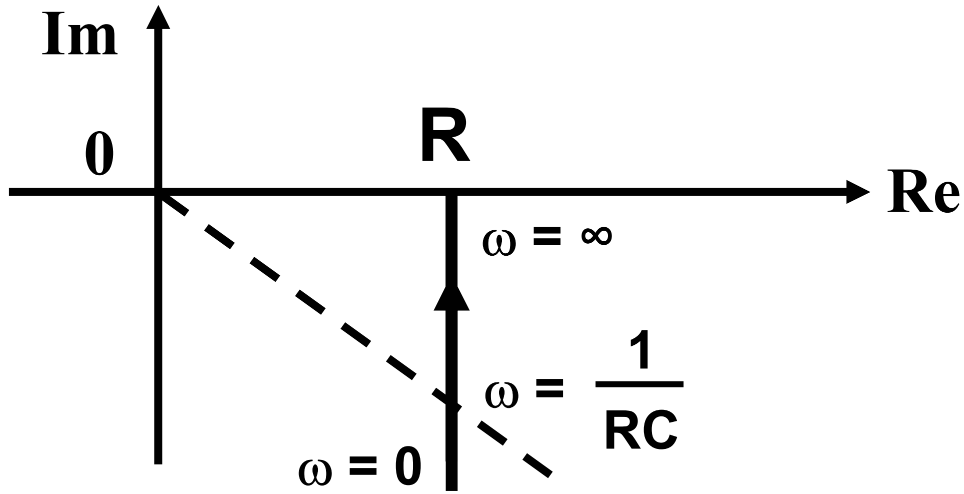


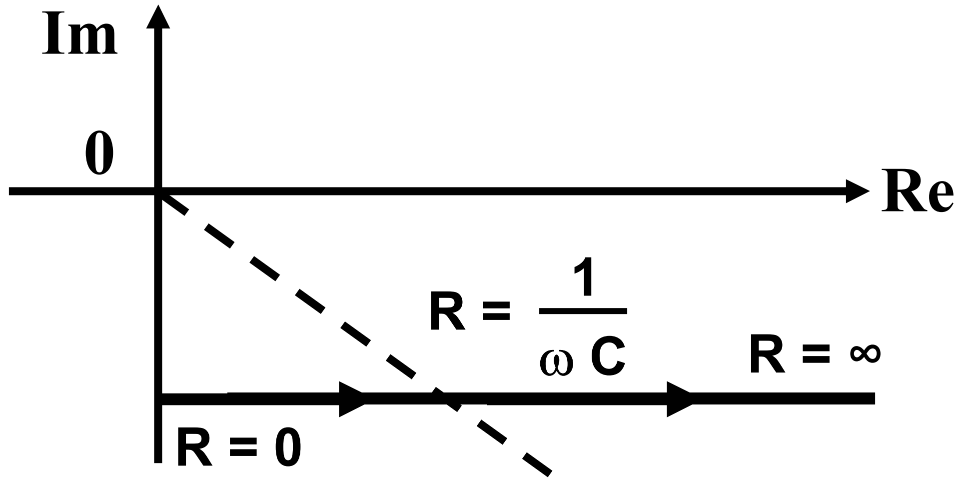


Polar diagram of RC network - 2

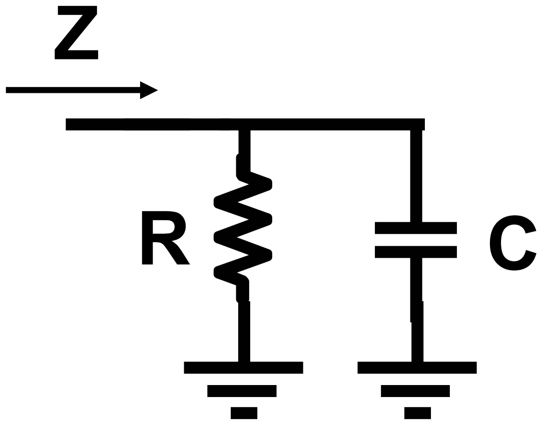


$$Z = R + \frac{1}{Cj\omega}$$

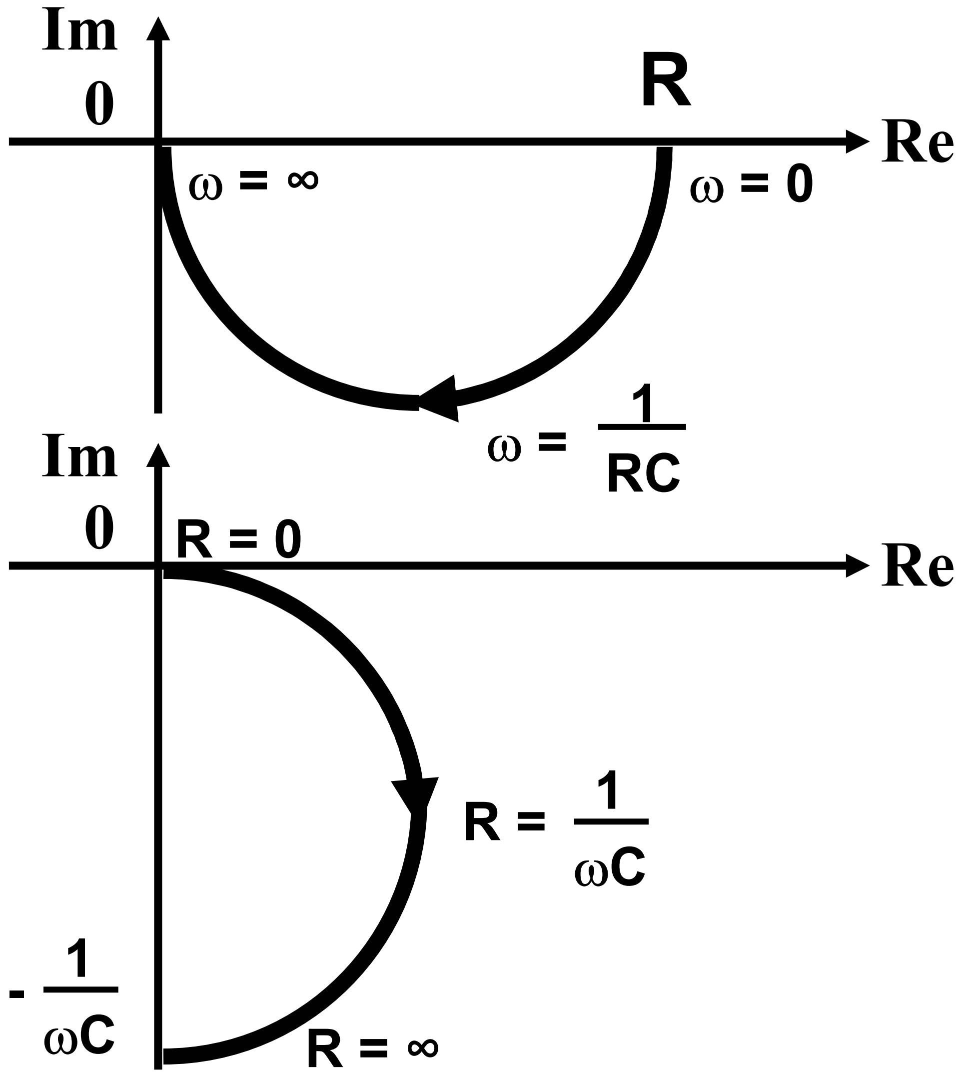




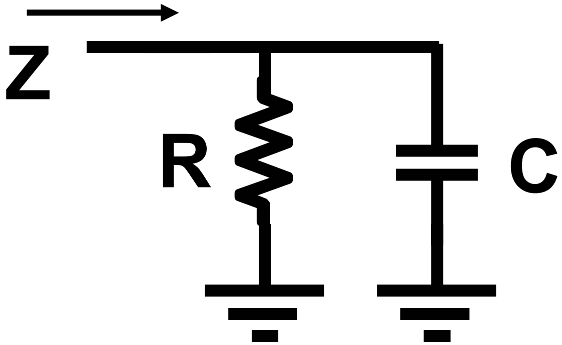
Polar diagram of RC network - 3

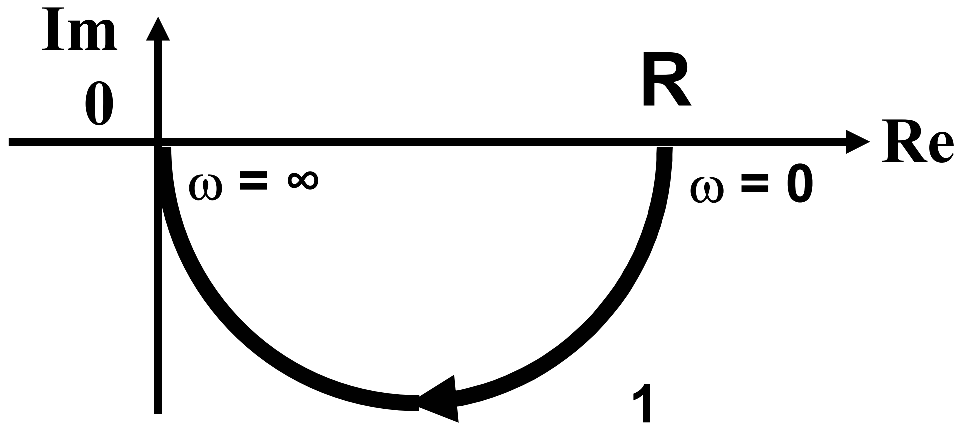


$$Z = \frac{R}{1 + RCj\omega}$$

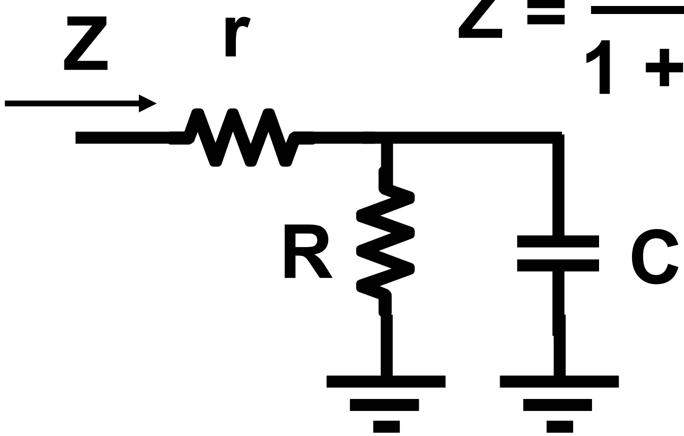


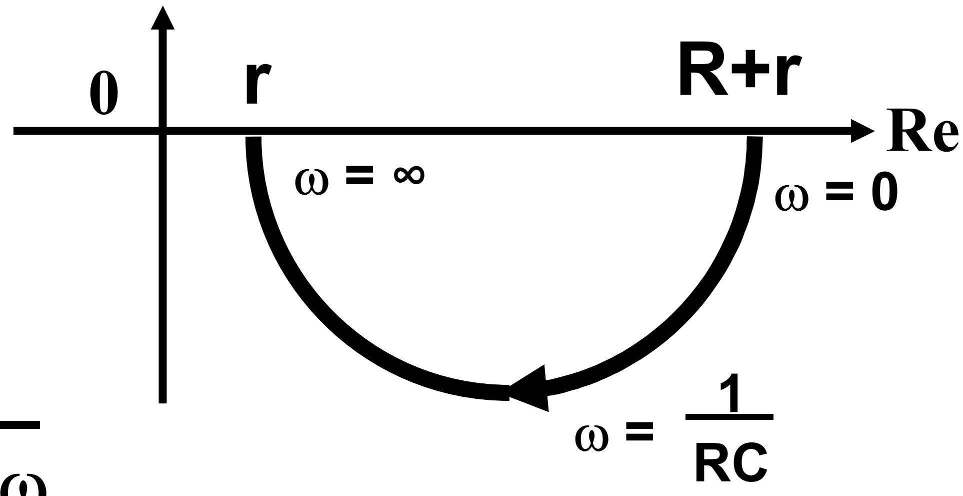
Polar diagram of RC network - 4





$$Z = \frac{R}{1 + RCj\omega}$$

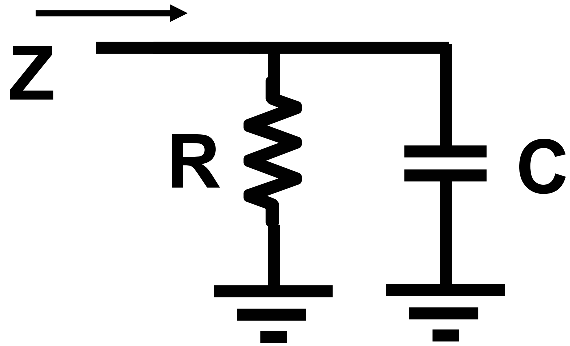




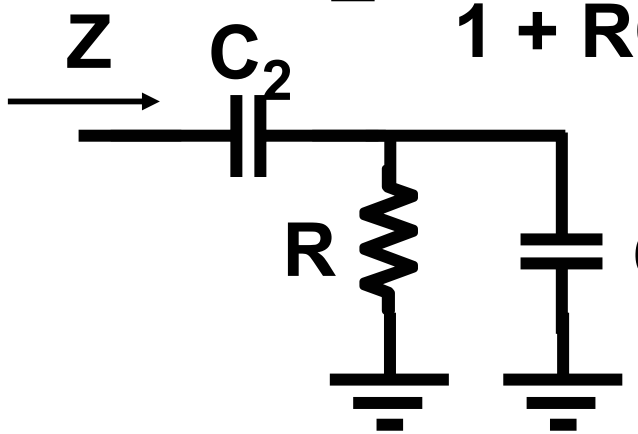
$$Z = r + \frac{R}{1 + RCj\omega}$$

Ref. Sansen, JSSC Dec.72, 492-498

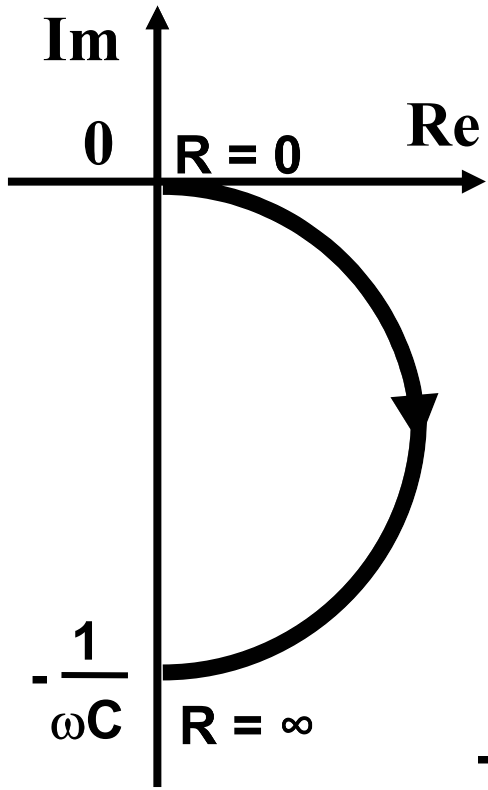
Polar diagram of RC network - 5

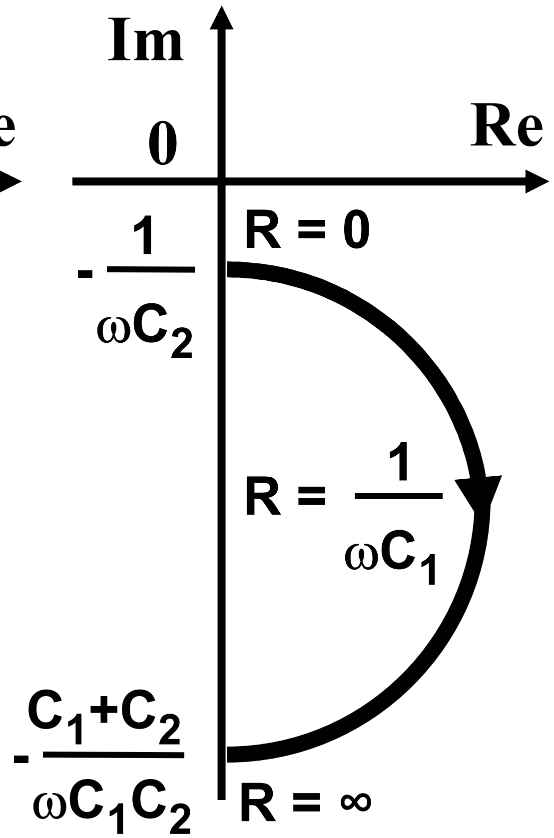


$$Z = \frac{R}{1 + RCj\omega}$$



$$Z = \frac{1}{j\omega C_2} + \frac{R}{1 + RC_1 j\omega}$$

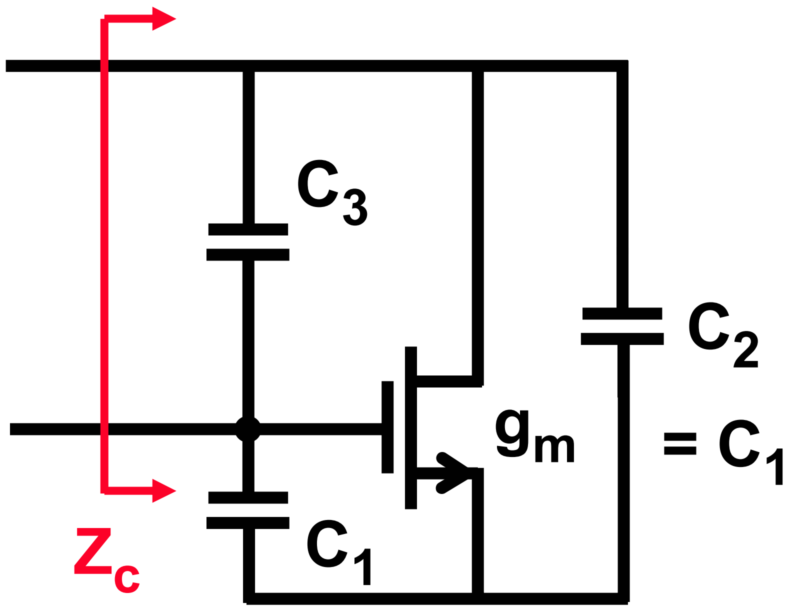




Circuit input impedance Zc

$$Z_c \approx \frac{g_m + 2\,j\omega C_1}{j\omega C_3\ (g_m + \frac{C_1}{C_3}\,j\omega C_1)}$$

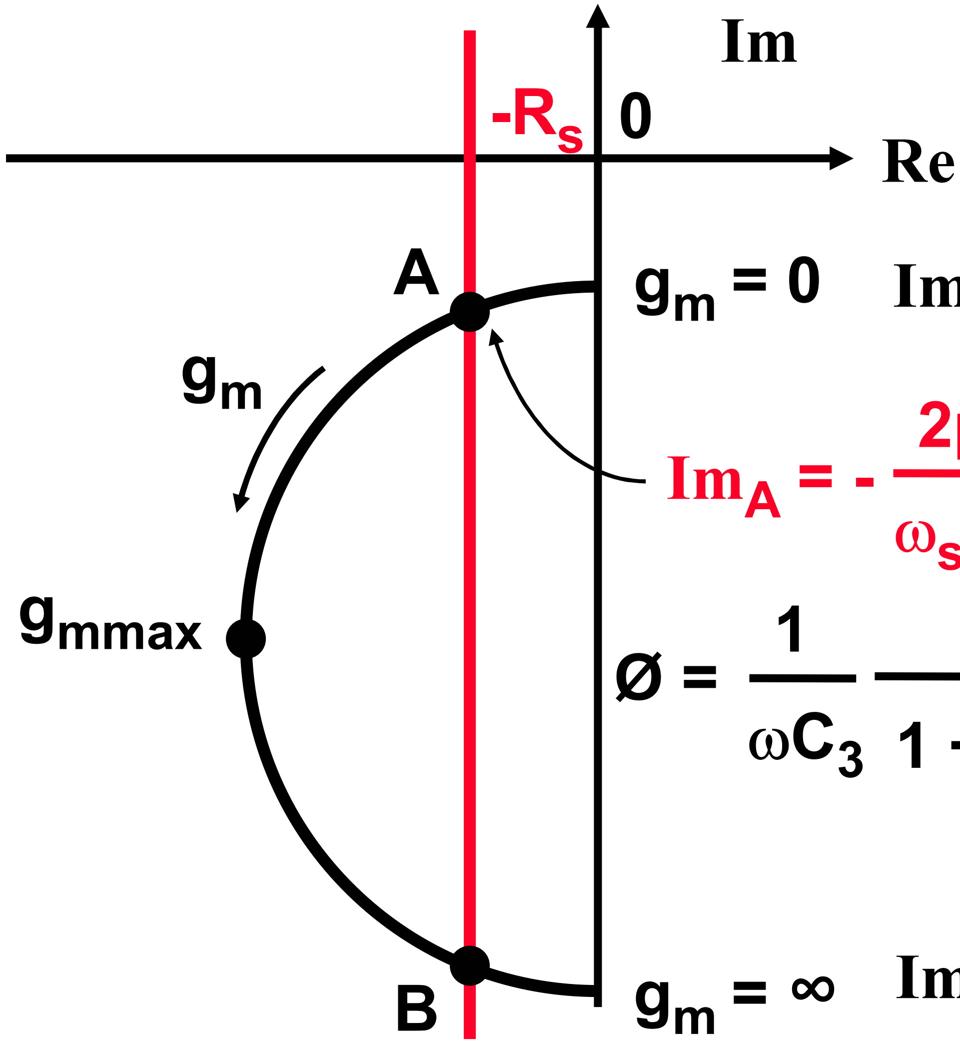
if $C_3 << C_1 = C_2$

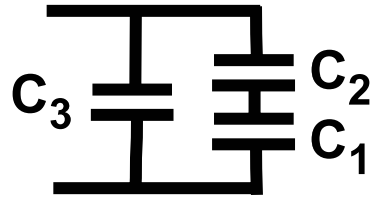


For $g_m \approx 0$ $\quad Z_{c0} \approx 2/\omega C_1$

For $g_m \approx \infty$ $\quad Z_{c\infty} \approx 1/\omega C_3$

Complex plane for 3-point oscillator



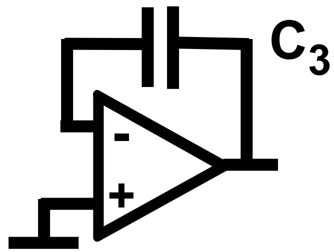


$g_m = 0 \quad Im_0 \approx - \dfrac{1}{\omega\left(C_3 + \dfrac{C_1 C_2}{C_1 + C_2}\right)}$

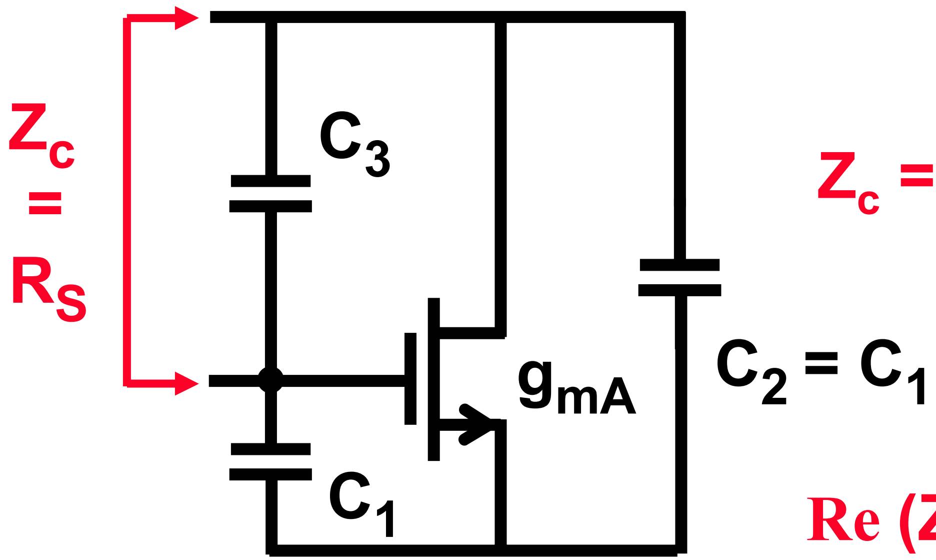
$Im_A = - \dfrac{2p}{\omega_s C_s} \approx Im_0$

$$Ø = \frac{1}{\omega C_3} \, \frac{1}{1 + C_3 \dfrac{C_1 + C_2}{C_1 C_2}}$$

$g_m = \infty \quad Im_\infty \approx - \dfrac{1}{\omega C_3}$



Calculation of g_{mA}



$$Z_c = \frac{1}{C_3 s} \frac{g_m + (C_1+C_2)s}{g_m + \left(C_1+C_2+\frac{C_1C_2}{C_3}\right)s}$$

$$Re\,(Z_c) = R_s$$

For small g_m : $g_{mA} \approx R_s\,(C_{eff}\,\omega_s)^2$

$$C_{eff} = C_1\left(1+\frac{2C_3}{C_1}\right) \approx C_1$$

Maximum negative resistance is $1/\,2\omega C_3$ at $g_{mmax} = \frac{C_1}{C_3}\,\omega C_1$